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PREDICTABILITY LIMITATIONS OF VIBRATION TRANSFER FUNCTIONS FOR STRUCTURES WITH OVERLAPPING MODES

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ABSTRACT

Large variations in noise and vibration transfer functions are usually observed between nominally identical products e.g. cars, aircraft, household machinery etc. This variation is usually blamed on insufficient manufacturing quality. It is more relevant to divide the variability in two components. One is due to allowed tolerances of material properties, geometry and dynamic properties of joints. The other is due to more considerable parameter variations for specific parts or joints as a result of unacceptable production quality.

Manufacturing tolerances of components and assembly of a product sets a basic predictability limit for dynamic transfer functions. It also defines a practical limit of accuracy for model updating, as it is a "designed" variability of the product population. The paper demonstrates variability mechanisms for frequency response functions (FRFs) of multi-modal systems. The considerable increase in FRF variance when modes are overlapping is illustrated by examples using modal models of simple systems. Variability of point-point transfer functions and of response energy levels are shown. The prediction capability and suitability of statistical energy-flow methods versus detailed FEM/BEM methods for frequency regions with different degrees of modal overlap is discussed.

1. BACKGROUND

It is well known that significant variations in vibro-acoustic transfer properties are obtained between individual products that are produced to be identical. Variations in the order of ± 5-8 dB for narrow-band transfer functions are usual and typical in serial production of road vehicles, aircraft, ships, appliances etc. at medium and higher frequencies.

A number of papers have been published on the variation of transfer function characteristics between road vehicles with nominally identical design [1]-[3]. Kompella et. al. [1] presented measured frequency response functions for a large number of identical vehicles. The FRFs show more random behaviour and larger scatter as the frequency increases, see Figure 1.
This scatter is often considered to be due to tolerances in assembly or low quality of supplied components, and QA-programs are introduced with the aim of reducing this variability. By assuming that the variability is mainly due to "insufficient manufacturing quality", also the use of large and highly detailed FE-models seems to be justified for prediction of dynamic properties, even at frequencies where many modes contribute significantly to the total response.

When the measured scatter is considered, the relevance of detailed predictions with deterministic methods can be questioned if they are applied at frequencies with significant modal overlap. Time and computer resources for creation and experimental updating of these models need to be optimised with respect to the achievable prediction accuracy. Statistical energy methods (SEA, EFA etc.) should be considered as efficient alternatives for medium and high frequency, leading to a considerably lower effort for modelling and computations.

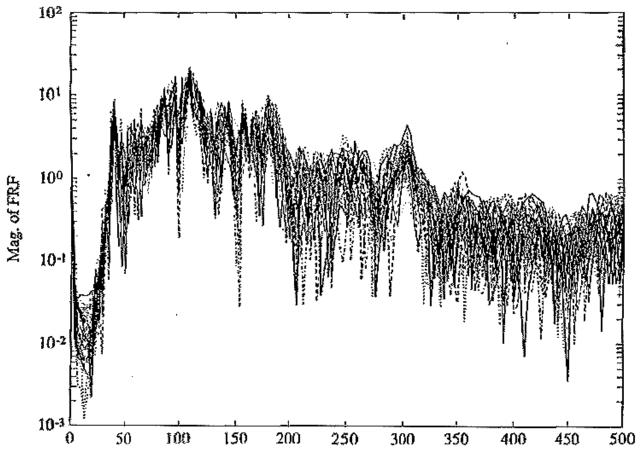

Fig. 1: Magnitudes of the 99 structure-borne FRFs for the RODEOs for the driver microphone [1].

This paper will not address the variability that may be caused by actual quality or tolerance problems in the manufacturing process of components or assembled products. Instead the fundamental limitations of deterministic prediction of dynamic response for multi-modal systems caused by input parameter uncertainty are shown.
A good summary of the fundamental limiting factors for deterministic modelling and analysis was presented in a recent SEA review paper by Fahy [4]. Apart from the well accepted fact that FE- or BE-models become very large at higher frequency due to the finer meshing requirements and that the modelling effort is increasing

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substantially due to more attention to geometrical detail, the following fundamental limits for prediction of the response in detail exist:

- *Uncertainty about precise dynamic properties*. Sensitivity of eigenfrequencies and phase response to changes in boundary conditions, thickness- and damping distribution etc. increases with mode order.
- *Modal summation.* Contributions from an increasing number of modes are added at each frequency as frequency and/or damping increases.
- *Uncertain dynamic properties of joints*. The dynamic force transmission properties of joints are not very well defined. In addition, dynamic properties of most joints between structural components are especially uncertain at higher frequencies.
- *Uncertain material properties.* Use of alloys, polymers and composite materials makes basic material properties considerably harder to predict for modelling purposes. In addition these material properties will vary much more due to temperature, static loads etc.
- *Uncertain modal damping estimation.* Forced response prediction needs damping to be estimated. For detailed deterministic prediction, either the correct spatial damping distribution or the correct estimates for individual modal damping has to be applied.

It is therefore essential to treat response prediction for multi-modal systems as a probabilistic problem. High-frequency response of a population of nominally similar products of which the individual members differ in many unpredictable details may be described by an ensemble-average behaviour, together with statistical estimation of the distribution of responses around this average.

One may randomise parameters and properties using some assumed distributions and deterministic computations to each member of the set so generated. This is impractical due to cost and time for large FE-models, but probably not even feasible because of problems to model multi-dimensional joint probability distributions for the large number of parameters. An alternative is using statistical energy methods (SEA) [4], [5].

2. THEORY

The statistics of multi-modal systems was first derived for room acoustics, see e.g., [7]-[9]. It has also been studied during the SEA development [10]. Schröder derived some fundamental results already in 1954 [7].
Consider a system where the response at each point and frequency is determined by the sum of a sufficient number of modes with random phase, and where no individual mode is dominating the sum. This will be the case in many dynamical systems above a certain frequency.

Define a logarithmic response function z as [7]

$$z = \ln \frac{x^2}{\overline{x^2}} \qquad (1)$$

where x is the response in the system at a certain point and frequency, and $\bar{x}$ is the spatial or frequency average value of x.

The standard deviation of z, $\sigma = \sqrt{\overline{z^2} - \bar{z}^2}$ can be calculated if the probability distribution function $W(z)$ is known. The real and imaginary parts of the complex response function are subject to Gaussian distribution

$$W(\mathrm{Re}(x)) = \frac{1}{\sqrt{2\pi\overline{\mathrm{Re}(x)^2}}} \mathrm{e}^{-\mathrm{Re}(x)^2 / 2\overline{\mathrm{Re}(x)^2}} \qquad (2)$$

and

$$W(\mathrm{Im}(x)) = \frac{1}{\sqrt{2\pi\overline{\mathrm{Im}(x)^2}}} \mathrm{e}^{-\mathrm{Im}(x)^2 / 2\overline{\mathrm{Im}(x)^2}} \qquad (3)$$

where $\overline{\mathrm{Re}(x)^2} = \overline{\mathrm{Im}(x)^2}$.

The Gaussian distribution is valid when the response is given as the sum of several complex independent (modal) vectors, of which no one is dominating. This type of response summation is illustrated in Figure 2.

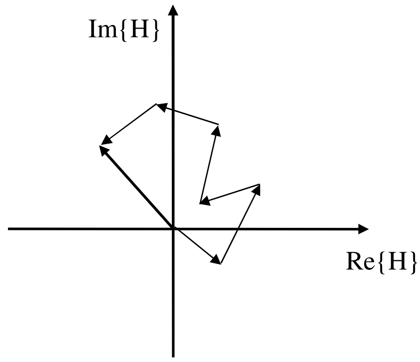


Fig. 2: The complex FRF and its modal components, for one frequency. No mode dominates.

Figure 3 illustrates a frequency response function with one dominating component, e.g., a dominating mode or the direct wave field of a strongly damped system. For this case the Gaussian distribution of real and imaginary parts does not apply.

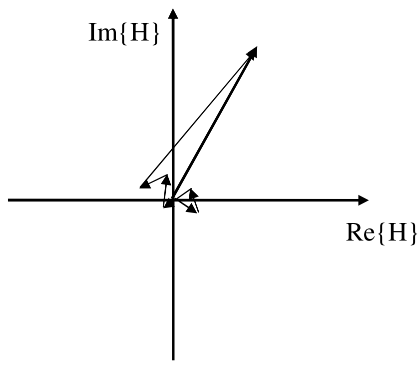


Fig.3: The complex FRF and its modal components, when one component is dominating.

The probability distribution function for z, $W(z)$ can now be derived as

$$W(z) = \exp(z - e^{z}) \quad (4)$$

with z from Equ. (1).

The standard deviation $\sigma(z)$ for this distribution is [7]

$$\sigma(z) = \sqrt{\overline{z^2} - \bar{z}^2} = 1.28 \quad \text{(Nepers)} \quad (5)$$

This standard deviation corresponds to $\underline{\sigma = 5.57\ \text{dB}}$.

3. GENERIC VIBRO-ACOUSTIC MODELS

The theoretical variance given above was derived from a general model involving summation of complex vectors. It should therefore be valid for any dynamic system for which the frequency response function can be expressed as such a sum. For any system with N vibrational modes, the FRF between two points can be expressed as

$$H(\boldsymbol{x}, \boldsymbol{x}_e, \omega) = \frac{\bar{v}(\boldsymbol{x}, \omega)}{F(\boldsymbol{x}_e, \omega)} = \frac{4\,\mathrm{j}\omega}{M} \sum_{i=1}^{N} \frac{\phi_i(\boldsymbol{x}) \cdot \phi_i(\boldsymbol{x}_e)}{\omega_i^2 (1 + \mathrm{j}2\zeta_i) - \omega^2} \quad (6)$$

where $\boldsymbol{x}$ is the spatial vector, $\boldsymbol{x} = [x,y,z]$, index e refers to excitation point, M is total mass, ω is excitation frequency and ϕ_i is the eigenfunction of mode i.

A completely generic multi-modal dynamic system can be represented by a sum of "modes" according to Equ. (6) where the eigenfrequencies are distributed as

$$\omega_i = 200\pi \cdot \log(i + 1) \quad (7)$$

and the eigenfunctions at excitation and response positions are random numbers between -1 and 1.
Random shifts in eigenfrequencies and damping for the individual modes are introduced to simulate the effect of material and geometric parameter variations for real structures. The eigenfrequencies are shifted as follows:

$$\omega_{ij} = \omega_{ij0} \cdot (1 + \varepsilon U) \ \text{or} \ \omega_{ij} = \omega_{ij0} \cdot (1 + \varepsilon U_{ij}) \quad (8a, b)$$

where ω_{ij0} is the unshifted eigenfrequency, ε is the amplitude of the random variation and U and U_{ij} are random numbers with normal distribution (m=0, σ=1).

For *global* parameter variations (e.g. average plate density or modulus) eigenfrequencies are shifted with the same relative frequency εU (Equ. 8a). Examples are given in [6]. *Local* variations of thickness, mass, boundary conditions etc. results in individual shifts in eigenfrequency for each mode (Equ. 8b), where each ω_{ij} is shifted by εU_{ij}. An ensemble of plates is obtained by using different sets of samples U_{ij}.

The modal damping has the same nominal value, ζ_{ij0}, for all modes. The uncertainty in damping is modelled by an exponential normal distribution, see equation (9). U_{ij} has a normal distribution with an mean value of 0 and a standard deviation of 1. An exponential normal distribution is chosen as it provides a realistic damping distribution for the modes.

$$\xi_{ij} = \xi_{ij0} \cdot 10^{\varepsilon U_{ij}} \quad (9)$$

Figure 4 shows an example of the difference in FRF that is obtained for two samples of the generic model.

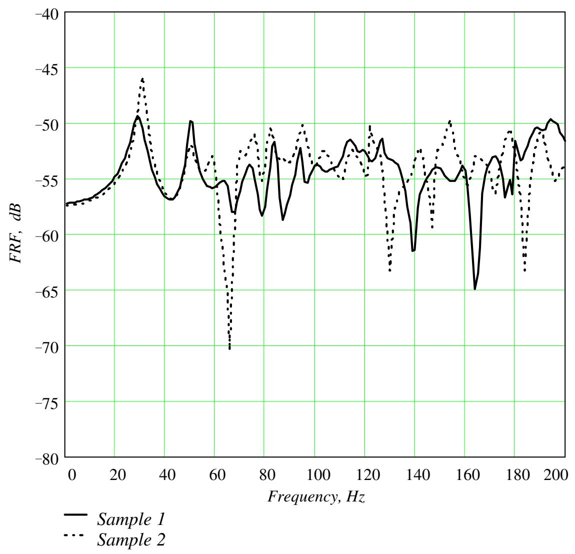


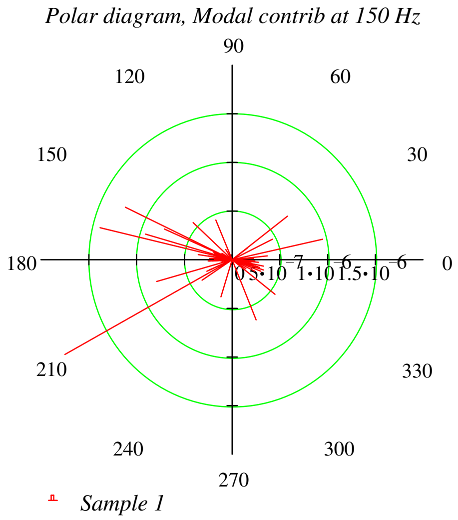
Fig. 4: FRFs for the generic modal expansion model with η = 5%. Standard deviations: 3% for eigenfrequencies, 30% for logarithm of the modal damping.

The modal overlap factor, which is defined as

$$MOF = n(f)\eta f \quad (10)$$

where *n(f)* is the average modal density (modes/Hz) and η the loss factor at frequency *f*, is larger than 1 for $f >$ 100 Hz in this case. Schröder's formulation is applicable when the modal overlap factor is larger than 2-3. When the modes have approximately equal excitation no single mode dominates the response in that case and the response is determined by a sum of several modes with different phase and amplitude.

The complex vector contributions from individual modes are shown for 150 Hz in Figure 5 a and b for the two samples. The large differences in phase and amplitude of modal contibution vectors that lead to the 3-4 dB FRF difference in Figure 4 are obvious.



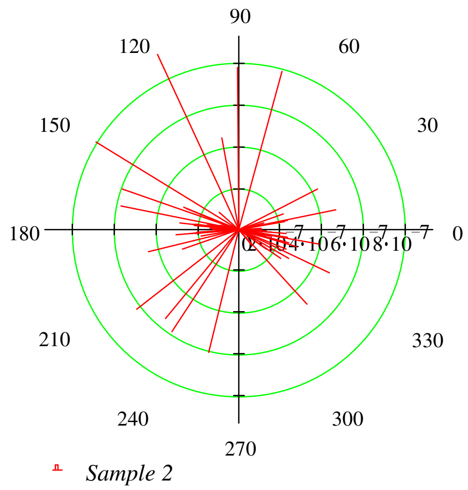


Fig. 5: Modal contributions at 150 Hz for the two samples from the generic modal expansion model.

A thin rectangular plate was used to exemplify a real multi-modal component [6]. The dimensions of the plate, given in table 1, are typical for thin steel or aluminium stiffened plate systems. The plate has simply supported boundaries and is excited at one point. Frequency response functions are calculated between single points with a frequency resolution of 1 Hz.

	Plate
l_x [m]	0.6
l_y [m]	1.67
l_z [m]	10^{-3}
c_0 [m/s]	-
E [Pa]	$2.10 \cdot 10^{11}$
ν [-]	0.3
ρ [kg/m^3]	7850
ξ [-]	0.03

Table 1: Input parameters for the rectangular plate

For the plate the FRFs are given by

$$H(\boldsymbol{x},\boldsymbol{x}_e,\omega) = \frac{\bar{v}(\boldsymbol{x},\omega)}{F(\boldsymbol{x}_e,\omega)} = \frac{4\,\mathrm{j}\omega}{\rho h A}\sum_{\substack{i=1\\j=1}}^{\infty}\frac{\phi_{ij}(\boldsymbol{x})\cdot\phi_{ij}(\boldsymbol{x}_e)}{\omega_{ij}^2(1+\mathrm{j}2\zeta_{ij})-\omega^2} \tag{11}$$

where

$$\phi_{ij}(\boldsymbol{x}) = \sin\frac{\pi i x}{l_x}\sin\frac{\pi j y}{l_y} \tag{12}$$

l_x, l_y are the lengths of the sides of the plate, A is the area of the plate, h is the plate thickness, ζ_{ijk} is the critical damping ratio for mode ijk.

The eigenfrequency ω_{ij} for mode ij is calculated using the following equation

$$\omega_{ij} = \sqrt{\frac{Eh^2}{12\rho(1-\nu^2)}}\left[\left(\frac{\pi i}{l_x}\right)^2+\left(\frac{\pi j}{l_y}\right)^2\right] \tag{13}$$

The model of the plate was used to calculate the variation of FRFs resulting from both global parameter variations in the plate and the variations of FRFs resulting from localized parameter variations [6]. Only some examples of the later will be shown below in this paper.

The excitation and response positions on the plate are the same for all plate samples although arbitrarily chosen. This means that the same point-point frequency response function is plotted for all plate samples.

4. NUMERIC EXAMPLES FOR LOCAL PARAMETER UNCERTAINTIES

Manufacturing processes like rolling, stamping, welding and moulding etc. will introduce localised variations in geometry, thickness, pre-stresses and possibly also material parameters. These local effects will shift individual eigenfrequencies differently depending on how the mode shapes relate to the localised variations of the structure. Other local mechanisms that introduce shifts in individual mode eigenfrequencies are boundary condition variations. These may be due to joint parameter fluctuations as well as variability of connected parts. All these can be represented by random shifts of individual eigenfrequency around the nominal value.

Also variations in individual modal damping factors, due to differences in boundary conditions, material and joint damping distribution, sound radiation, etc. can be quite large. Scatter in these factors will cause additional variability of the damping for individual modes. This means that the relations between modal damping factors will also vary between different samples of a plate. This is modelled as randomly distributed damping between the individual modes.

Damping has a significant influence on the FRF, since it will change the amplitudes of the complex mode contribution vectors. Random variation in damping for individual modes will cause largely varying transfer functions as shown in [6].

When the individual eigenfrequencies scatter randomly around their average values, the transfer functions will get very dispersed. This sensitivity to rather small eigenfrequency shifts is explained by the large FRF phase angle jump around the natural frequency of the single-degree-of-freedom system that represents each mode. Examples are given in [6].

We expect a combined scatter of damping and natural frequencies in real structures, and the total impact of these variations on the variance in the FRFs will be a superposition of the respective effects. The modelling of the variations and their cause has been explained earlier. The necessary variation of the input parameters of the rectangular plate to get randomly varying FRFs for modal overlap larger than 2-3 is, e.g., a 2% eigenfrequency variation combined with a 20% variation in the logarithm of damping for the studied plate, see Figure 6.

The result obtained, using the combined variations of individual, modal eigenfrequencies and damping shows a good qualitative agreement with the reported measured results for complete cars, compare Figure 1 with Figure 6.

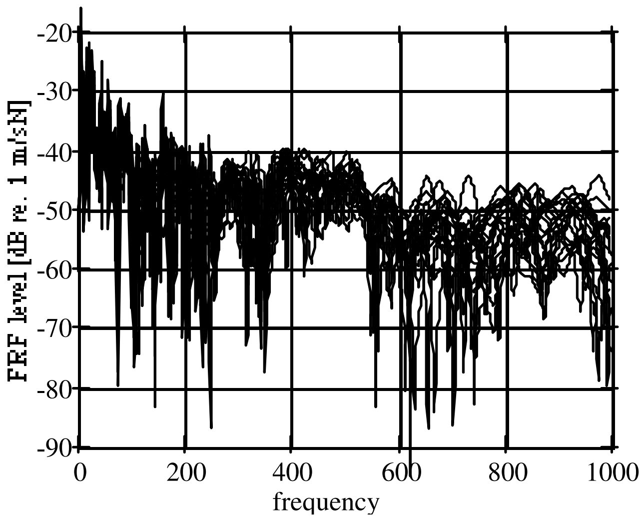


Fig. 6: FRFs for combined random variations of eigenfrequencies and modal damping. Standard deviations: 2% for eigenfrequencies and 20% for the logarithm of damping factor.

When energy methods (SEA) are used, spatial average responses for the subsystems are calculated [10]. Energy methods therefore seem to have the disadvantage of not calculating the responses for specific points in the system, but just the average value. However, as shown above, detailed FRF estimation is of a limited practical value, since quite small input parameter uncertainties will lead to low precision in the prediction anyway as soon as the modal overlap is significant.

On the other hand, the spatial average energy response is expected to fluctuate much less than the FRFs for corresponding variations in eigenfrequency and modal damping. To compare the sensitivity to these variations, the response of the same plate as before was calculated using energy methods with the analytical model developed by Fredö [13].

The sensitivity to individual shifts of eigenfrequencies, with a 2 % standard deviation, and an additional 20% standard deviation of the logarithm of the modal damping, was used as previously for the FRFs, see Figure 7. As expected, the energy response levels show much less variation than the FRFs. There is the same moderate dispersion between samples as for the FRFs at low frequencies. It is meaningful to predict responses from a detailed deterministic model in this frequency range, since it will reveal more detailed information about the structural response than a statistical energy model. However, at frequencies where modal overlap is significant, the average behaviour is predicted equally well by the energy model.

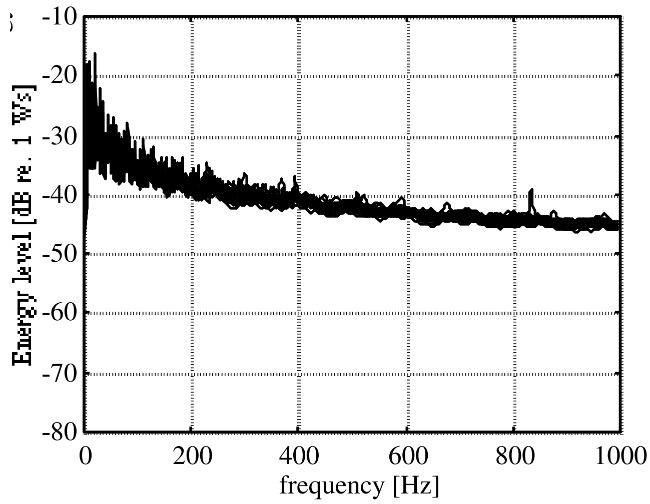


Fig. 7: Variation of spatial average velocity level (proportional to the kinetic energy) between different samples of the plate. Standard deviations: 2% for eigenfrequencies and 20% for logarithm of modal damping factors.

5. SCATTER IN BUILT-UP STRUCTURES

When a number of simple structural components (subsystems) are connected, it is expected that variations in the matching of the eigenfrequencies of local modes in the different subsystems eventually will make the FRFs between points on different subsystems to scatter more than FRFs between points on the component structures. The energy flow model developed by Fredö [13] for two connected plates was used for two connected plates in a L-configuration as illustrated in Figure 8. The parameters for the configuration are given in Table 2.

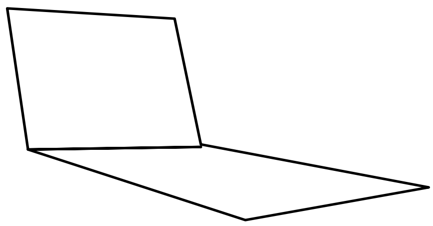


Fig. 8: The L-configuration of two simply supported plates used in the analytical model by Fredö [13].

The result of combined variations of individual modal eigenfrequency and damping, corresponding to the result shown in Figure 6 for the simple plate, is shown in Figure 9. The FRFs between an arbitrary point on plate 1 and a point on plate 2 is shown. The standard deviation in the frequency range with significant modal overlap reaches the same 5-6 dB value as for the single plate.

	Plate 1	Plate 2
l_{side} [m]	0.6	0.84
$l_{junct.}$[m]	1.67	1.67
h [m]	10^{-3}	10^{-3}
E [Pa]	$210 \cdot 10^{9}$	$210 \cdot 10^{9}$
ν [-]	0.3	0.3
ρ [kg/m^3]	7850	7850
ξ [-]	0.03	0.03

Table 2: Parameters of the L-plate configuration.

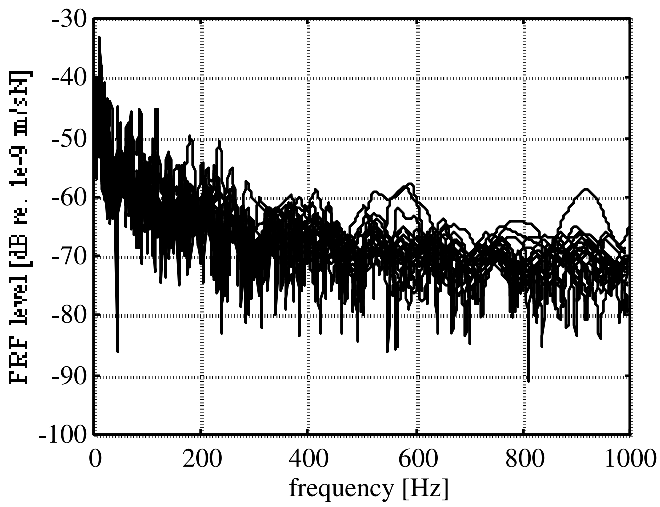


Fig. 9: Variation of FRF levels between points on two subsystems, calculated for the L-plate. Combined local parameter variations. tandard deviations: 2% for eigen-frequencies and 20% for logarithm of modal damping.

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The calculated response energy level in plate 2 with excitation in plate 1, corresponding to the result shown in Figure 7 for the single plate, is given in Figure 10.

The same response energy level has also been calculated with Statistical Energy Analysis (SEA), using the simple 2-plate model illustrated by Figure 11. The plates are represented as bending wave subsystems only. The model was created using the commercially available AutoSEA software. The SEA calculated result has been included in Figure 10 for comparison. As can be seen, the agreement with the exact analytical result is quite good. It can also be mentioned that the SEA modelling, computations and plotting of resulting energy levels or response levels were perfomed in about 5-10 minutes. Most of this time was spent on input of the subsystem data from Table 2 above.

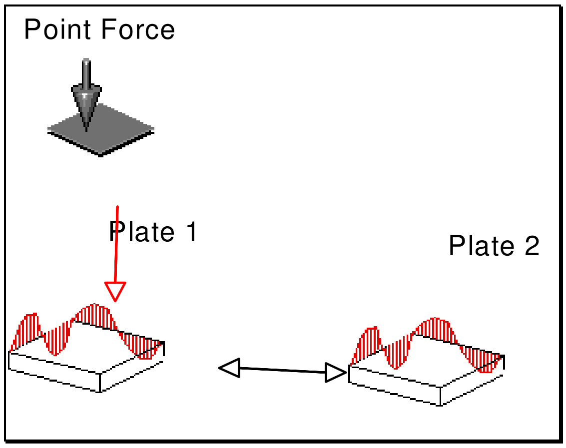

Fig. 11: SEA model of the L-plate configuration.

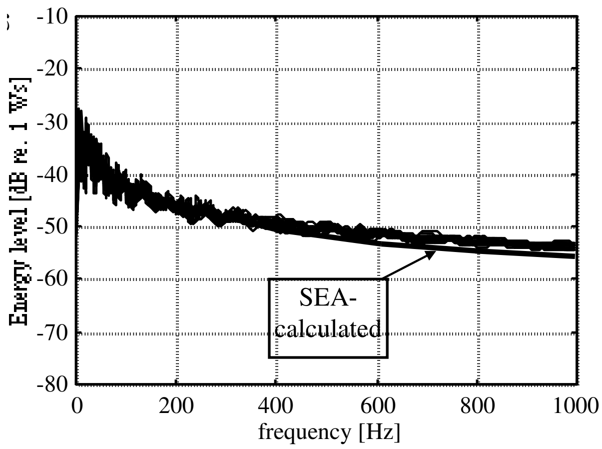

Figure 10: Variation of spatial average velocity level in the receiving plate of the L-plate. Combined local parameter variations with standard deviations: 2% for eigenfrequencies and 20% for logarithm of damping.

6. EXAMPLES OF THE INFLUENCE OF PRACTICAL PARAMETER TOLERANCES

Some work has been done by the author and his colleagues at Ingemansson as well as Chalmers on investigating actual component eigenfrequency and FRF variation. It has been carried out as parts of confidential industry related projects and details can not be quoted here. However, in general terms, even stamped, thin steel plate components without attached hardware will often show a noticable FRF standard deviation (1-3 dB) in the frequency range of modal overlap > 2-3. Considering the small manufacturing tolerances and a well controlled material, this is rather surprising.

The experimental investigations needed utmost care in order to obtain repeatable edge conditions and shaker attachment. These could influence the measured variability of the FRFs as well as the scatter in natural frequencies considerably.

For cast iron and cast aluminium as well as polymer components, the FRF standard deviation in the modal overlap region may approach the 5 dB upper limit for average production and material tolerances.

FE-models may be used to investigate how specific parameter tolerances influence the scatter of natural frequencies. One example of simulating local thickness (σ_t = 10%), Youngs modulus (σ_Y = 5%), and density (σ_d = 5%), variations for a plate is given in Figure 12. The calculations were made for a rectangular, simply supported plate using 300 triangular elements. The parameters have been varied randomly for 40 patches. The actual influence on natural frequencies when varying these parameters will start to diminish when the size of the "patches" become smaller than the "correlation length" of the mode, which is approximately half a bending wavelength.

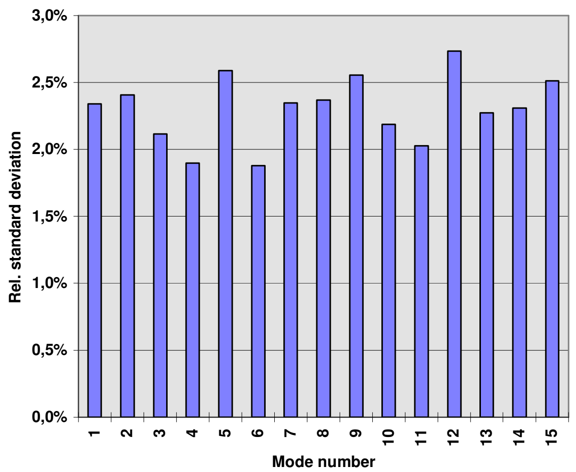

Fig. 12: Standard deviation for natural frequencies of different modes of the rectangular thin plate, obtained with FEM.

7. MATHEMATICAL MODELLING STRATEGY

Uncertain input data limit the quality of any prediction method. Detailed dynamic response functions for real products are only useful at frequencies where one or a few modes determine the response. Deterministic (FEM/BEM or analytical) modelling is best applied in this case.

The reliability of deterministic response prediction in road vehicles, aircraft, spacecraft etc. at medium and high frequency is not primarily determined by the size or the geometrical detail of a FE-model or even the modelling skill of the analyst. The limit is set by input parameter accuracy requirements as small variations in eigenfrequency and damping of individual modes will produce large FRF scatter due to overlapping modes. Updating of the FE/BE-model against hardware will not reduce this random error.

The needed input parameter accuracy will often exceed reasonable production tolerances, especially when polymer materials and assembly techniques. The additional product cost due to tighter tolerances and the corresponding QA-procedures can only be justified if it results in sufficient additional functionality or customer satisfaction and not only added satisfaction of the FE-analyst

FRFs for connected structures become even more sensitive to errors in input data. Small shifts in eigenfrequency and modal damping for "local" modes of subsystems may lead to large variation in mode-mode coupling functions at junctions. This variation is expected to be most important for low modal overlap.

Considerable modal overlap will occur in subsystems with high modal density like acoustic cavities, thin panels etc., already for moderate damping. Modal overlap will increase with increased damping. Spatial average response (energy) is the "stable" quantity describing the behaviour of the product population (ensemble) in frequency regions with high modal overlap. Statistical energy modelling (SEA) or other energy transmission prediction methods are usually more appropriate for this case.

The modal overlap factor is an easily estimated quantity to use for separating the two frequency regions.

8. CONCLUSIONS

It has been demonstrated that frequency response functions for simple substructures with overlapping modes are quite sensitive to rather small variations in local input parameters. This sensitivity may become even larger for built-up structures. Numerical simulation using realistic parameter tolerances show that the FRFs may become well randomised already when the modal overlap factor reaches about 2-3. Experimentally obtained variability by the author and published by other workers for nominally identical products like cars compare well with the simulations presented here and in [6].

The standard deviation for FRFs between specific points at a specific frequency is considerable. The variance is substantially smaller for spatial RMS-averages of the FRFs, which correspond to subsystem energy quantities, as they are used in existing statistical energy analysis (SEA) prediction.

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Statistical energy methods (e.g. SEA) for prediction is a serious alternative to deterministic modelling at medium and high frequency for many products. The modelling effort and computing resources are much less. The results also represent the average response for a random ensemble of structures. Result from a deterministic model may erroneously be interpreted as accurately representing the entire ensemble of products, especially if the model has been carefully updated.

Statistical energy analysis methods need to be developed further. Also the predictability issues as presented in this paper should be investigated in more detail also for more complex built-up structures. Successful and realistic computer aided design of dynamical properties for products the medium and high frequency range will rely on progress in these areas.

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