# GSSEA-Light®

Introduction to the Statistical Energy Analysis Method

# Contents

- **Statistical Energy Analysis - History and Overview**
- **Fundamental concepts**
- **Predictability of frequency response functions for real products**
- **SEA - a formalism for well known applied acoustics**

# Statistical Energy Analysis

- Calculation of acoustic and vibratory response in complex systems at higher frequencies
  - from ca 50-100 Hz in ship acoustics
  - from ca 150-200 Hz in vehicle acoustics
- Details of no significance omitted
  - Model components are divided as main plates, shells beams and acoustic cavities
- Vibrational energy response calculated
  - for different wave type groups (subsystems) in the components is calculated for a number of vibrational power inputs

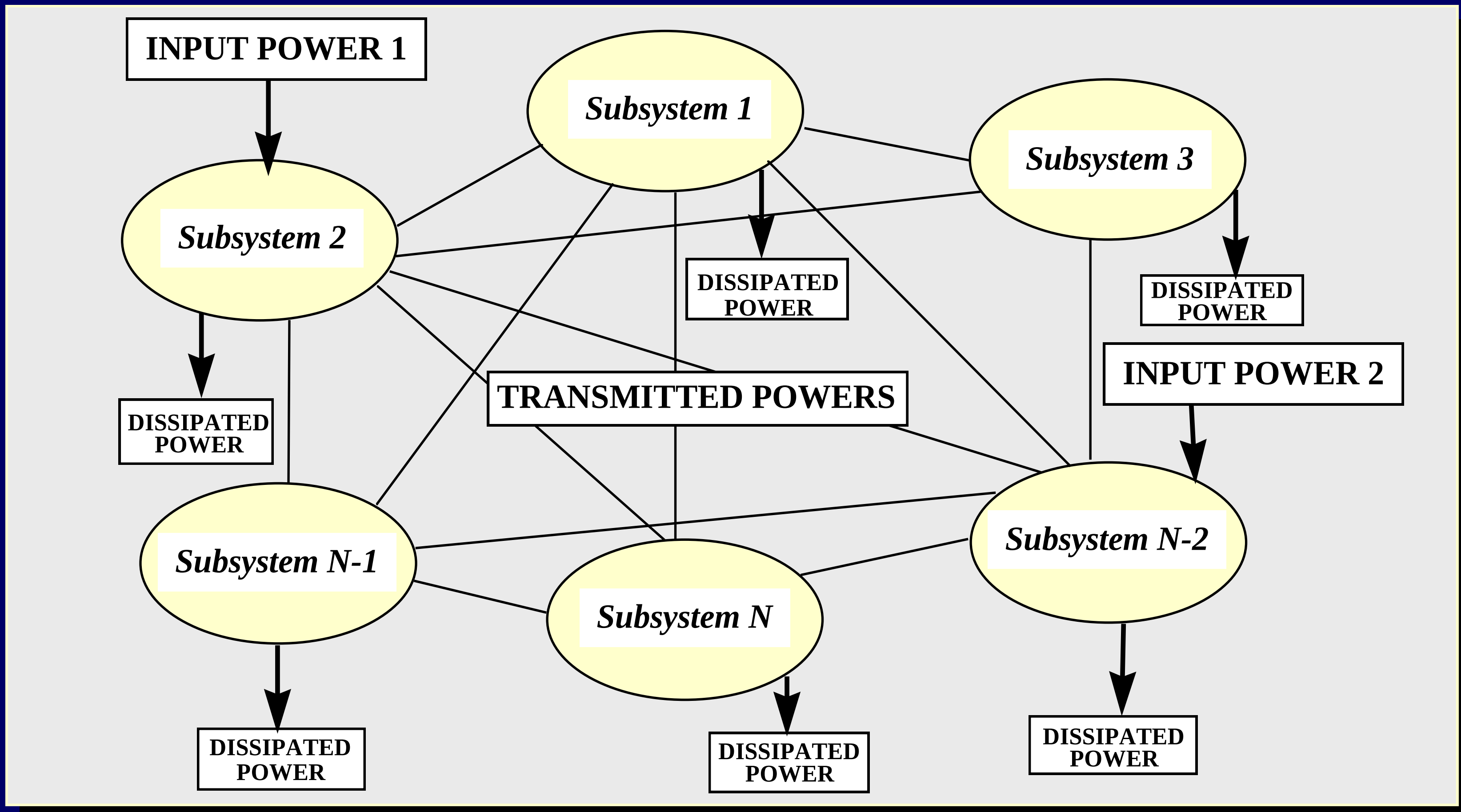
# SEA History

- **1950's**
  - New aero-engines and rockets need high intensity acoustic and vibration excitation design tools
- **1960's**
  - Random and modal vibration combines with statistical & wave room acoustics (Smith, Lyon & Maidanik, Sharton etc)
- **1970´s**
  - SEA application research outside aerospace, e g buildings, ships
- **1980's**
  - The ”dark” decade for SEA. FEM promises to be the standard tool for structural and acoustical analysis

# SEA Basic Ideas

- **"STATISTICAL"**
  - The subsystems in a coupled system are considered to be random samples of a population of systems with
    - the same <u>main</u> properties (area, thickness, material etc.)
    - randomly distributed details (shape, edge conditions etc.)
- **"ENERGY"**
  - is the primary calculated response variable. Other response variables are derived from the subsystem energies
- **"ANALYSIS"**
  - Not an established, codified method. It´s an approach to find the influence of system parameters on vibration energy distribution

# SEA Modelling Principle



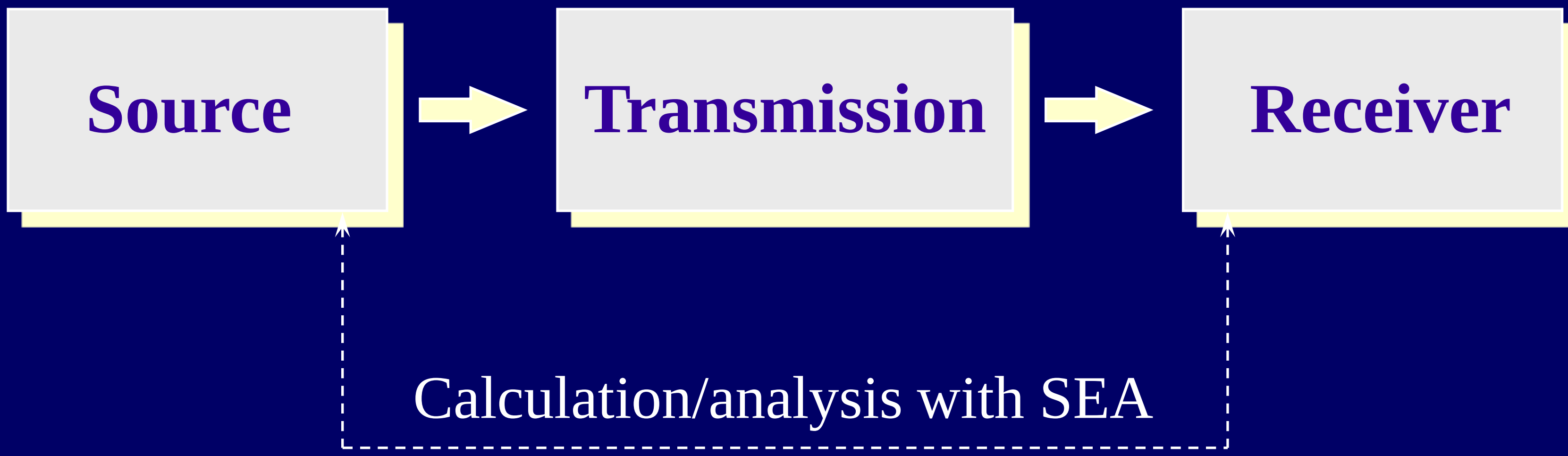
# Examples of applications

- **Aerospace (Lyon, Ungar etc from the 1960’s)**
- **Building acoustics (BBN, ISVR etc from the 1970’s)**
- **Naval, shipbuilding (Plunt, Ödegaard-Jensen etc from the 1970’s)**
- **Railway vehicles and track structures (Remington, Manning etc from the 1980’s)**
- **Automotive (deJong, Lalor, Moeller etc 1985- )**
- **Consumer appliances (1990’s)**
- **Nuclear engineering**

# Fundamental concepts

- **Transmission of sound and vibration**
- **Transfer functions-frequency response functions(FRFs)**
- **Oscillatory modes (natural modes)**
- **Free and forced vibrations (resonant-nonresonant)**
- **Mode superposition - Ex. Beam vibrations**
- **Modal overlap - MOF (*modal overlap factor*)**

# Transmission of sound and vibration



# Transfer functions - Frequency response functions (FRFs)


$$\underline{Y}(f) = \underline{H}(f) \cdot \underline{X}(f)$$

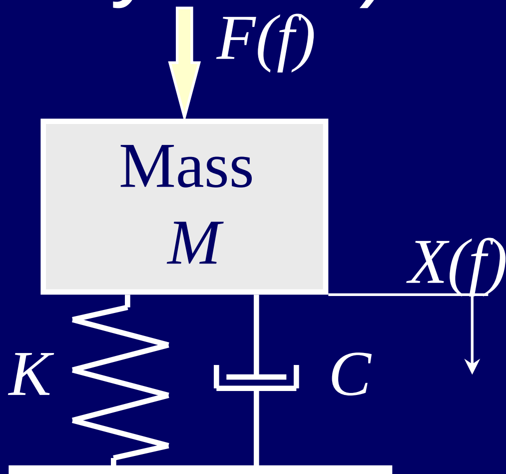
***H(f)*** is the transfer function (frequency response function-FRF)

***H(f)*** is a function with both *amplitude* and *phase*:

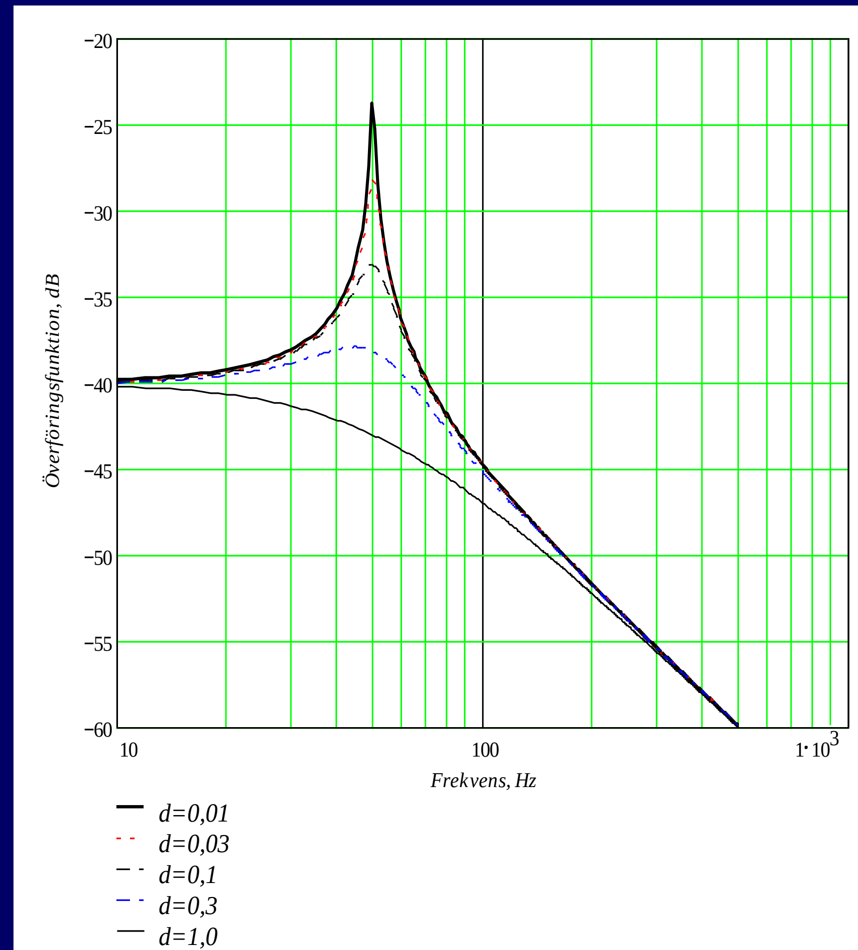
$$\underline{H}(f) = \left|H(f)\right| e^{j \arg(H(f))}$$

***H(f)*** also depends on boundary conditions - impedances $Z_k$ and $Z_m$

# Simple oscillator (mass-spring system)

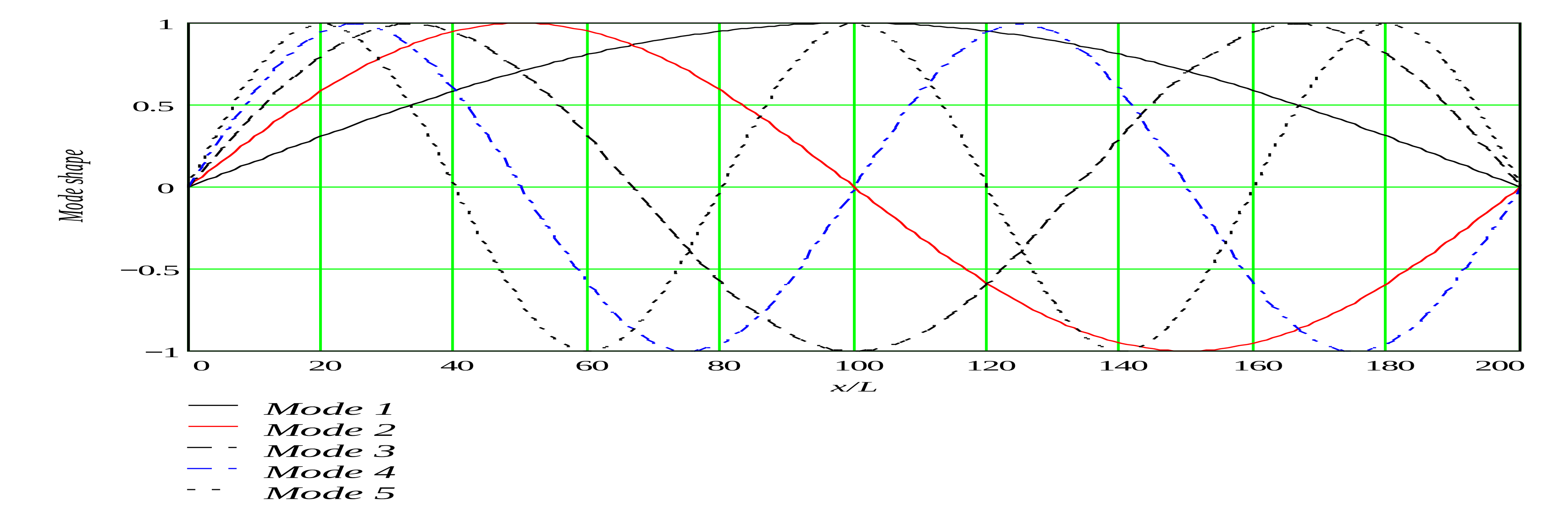



$$H(f) = \frac{X(f)}{F(f)} = \frac{1}{-\omega^2 M + j\omega C + K} = \frac{1}{K(1 - \frac{\omega^2}{\omega_n^2} + 2j\varsigma\frac{\omega}{\omega_n})}$$



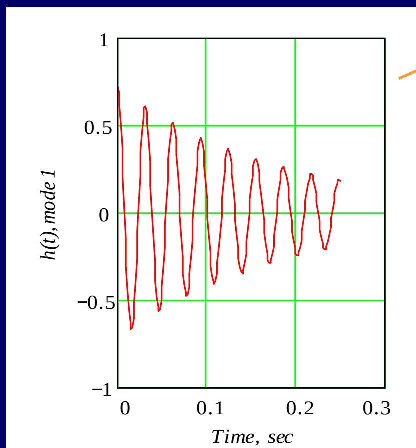
# Vibration modes - Beam example

Continuous mechanical systems have an infinite number of natural modes:

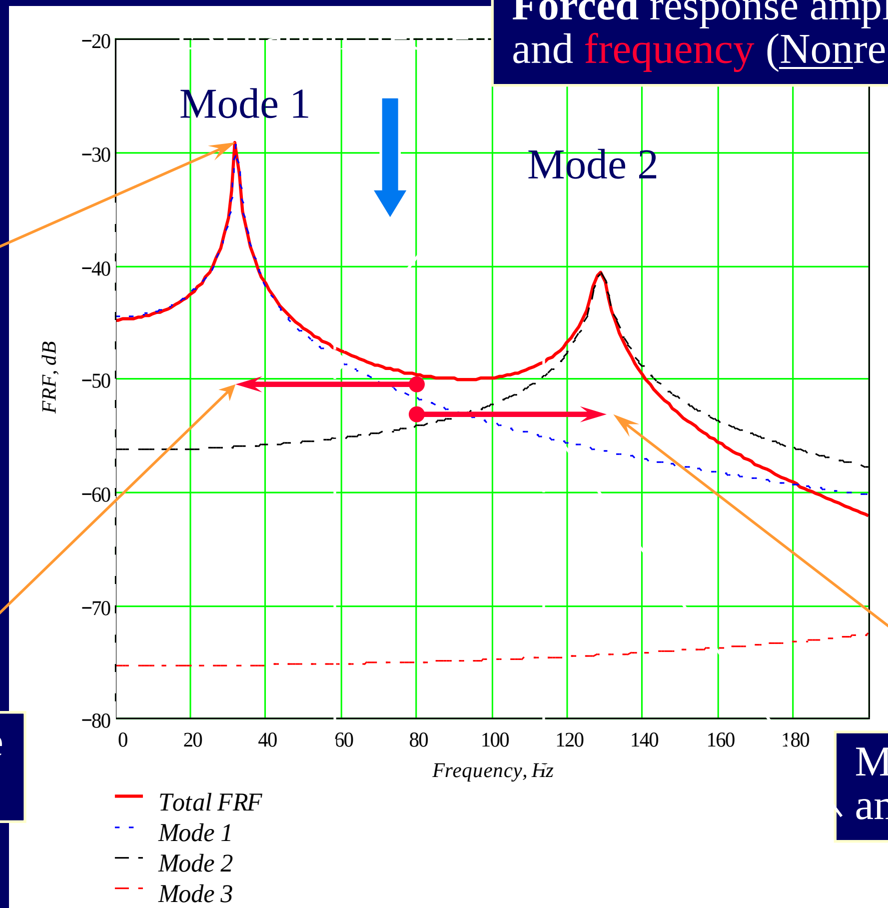


# Free and forced vibrations

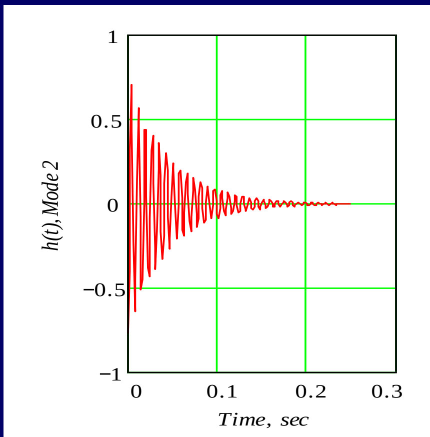
Free response
Mode 1




**Forced** response amplitude and frequency (Nonresonant)



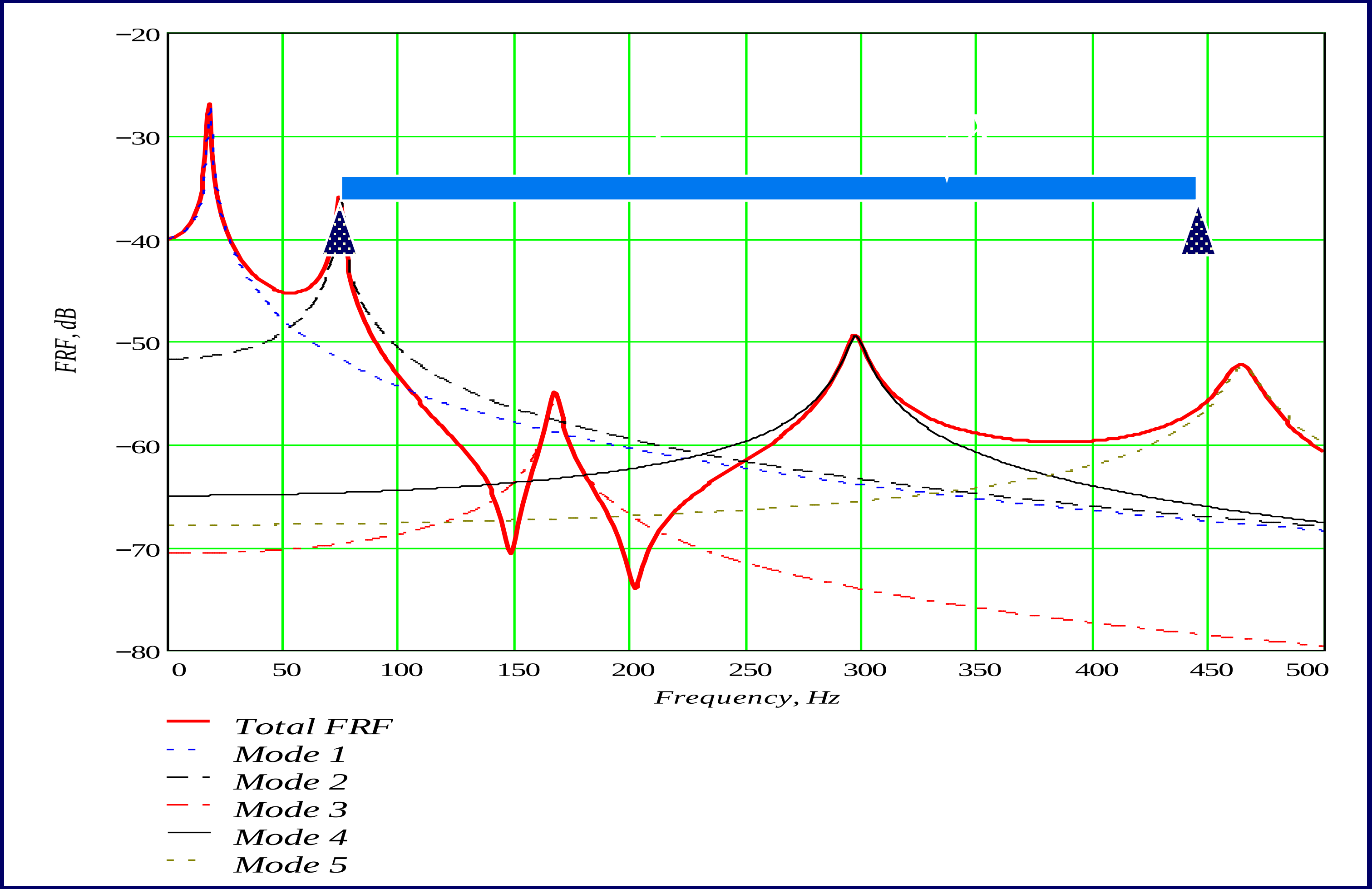

Free response
Mode 2




Mode 1 initial **free** response amplitude and frequency

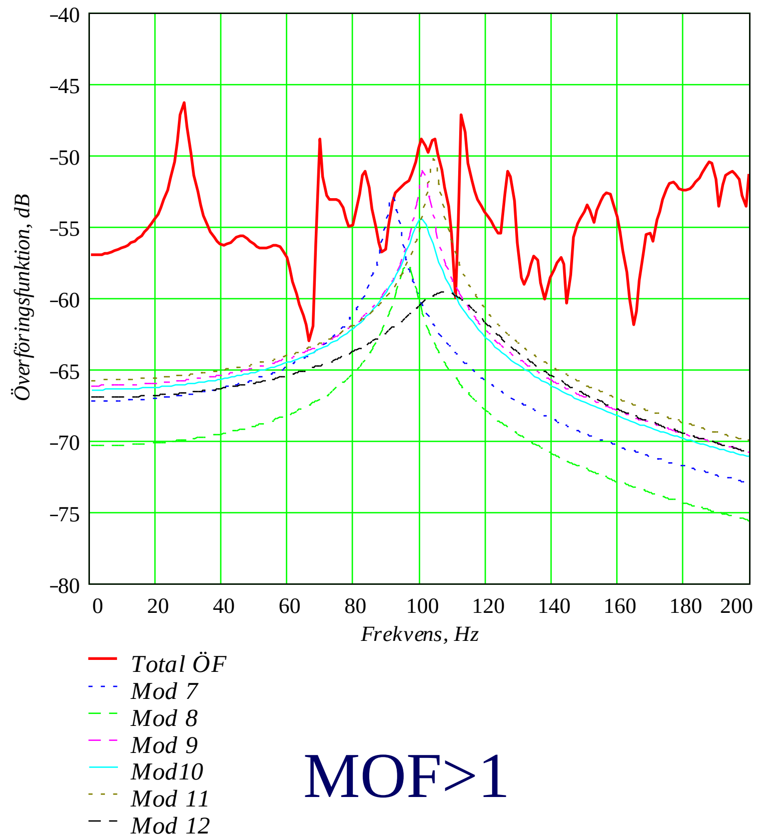
Mode 2 initial **free** response amplitude and frequency

# FRFs and modal superposition

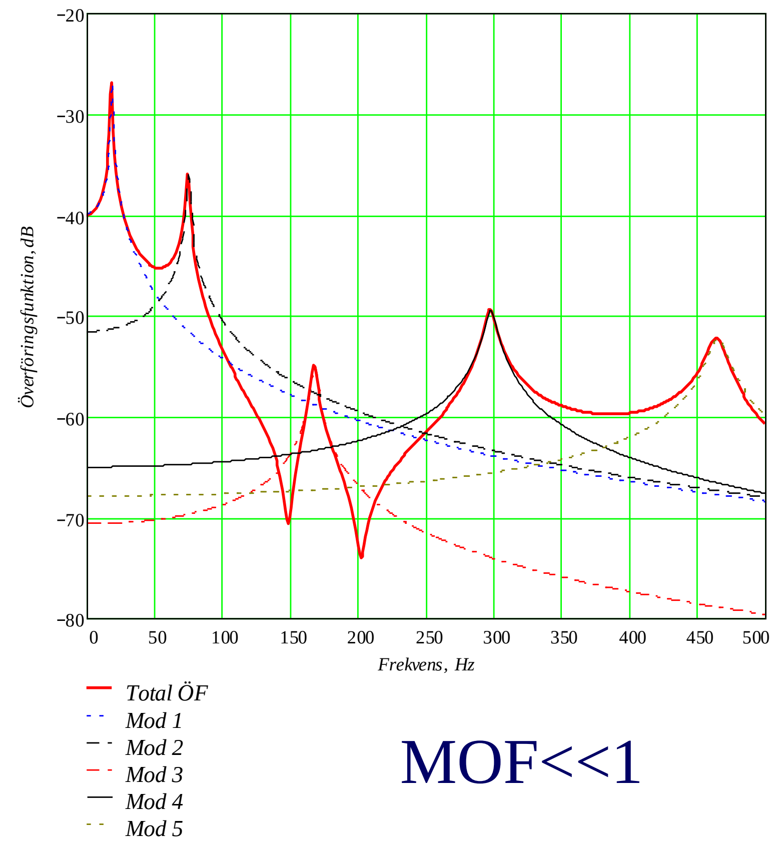


# Modal overlap (MOF)

Modal overlap factor: *MOF = Mode bandwidth*Modal density*

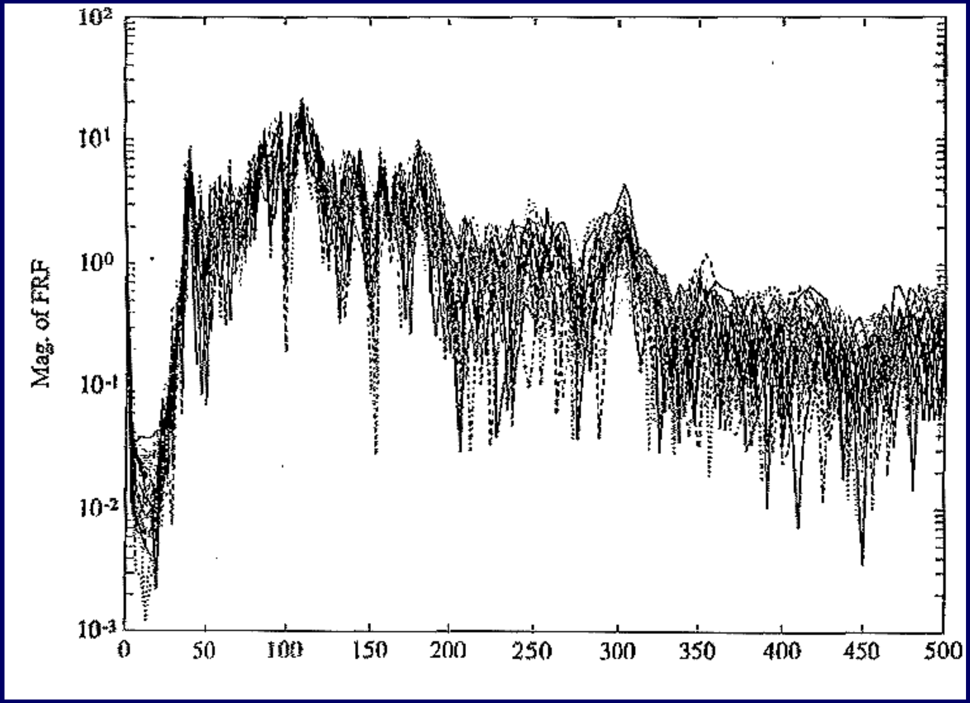



MOF>1




MOF<<1

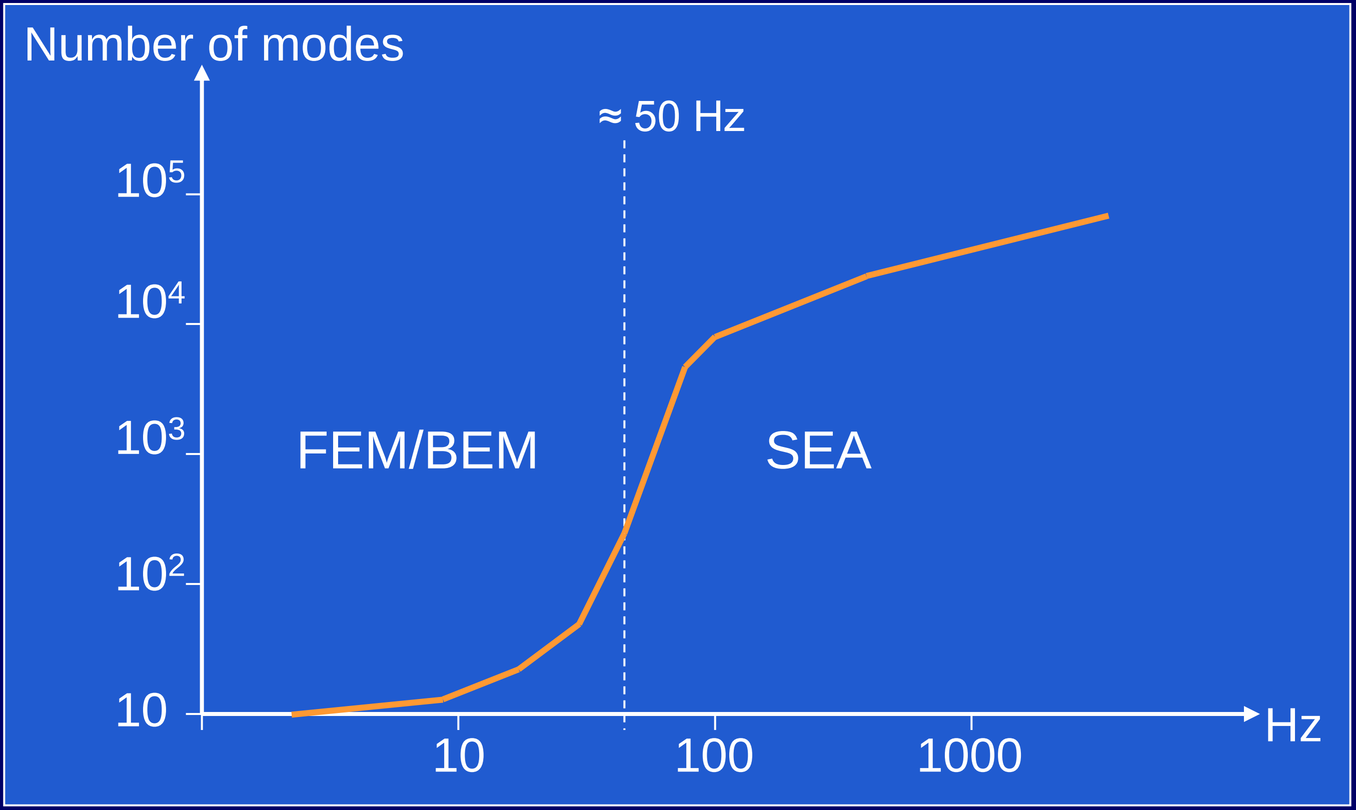
# Transfer functions – Practical variability




Transfer functions between wheel hub (F) and drivers ear position (p)

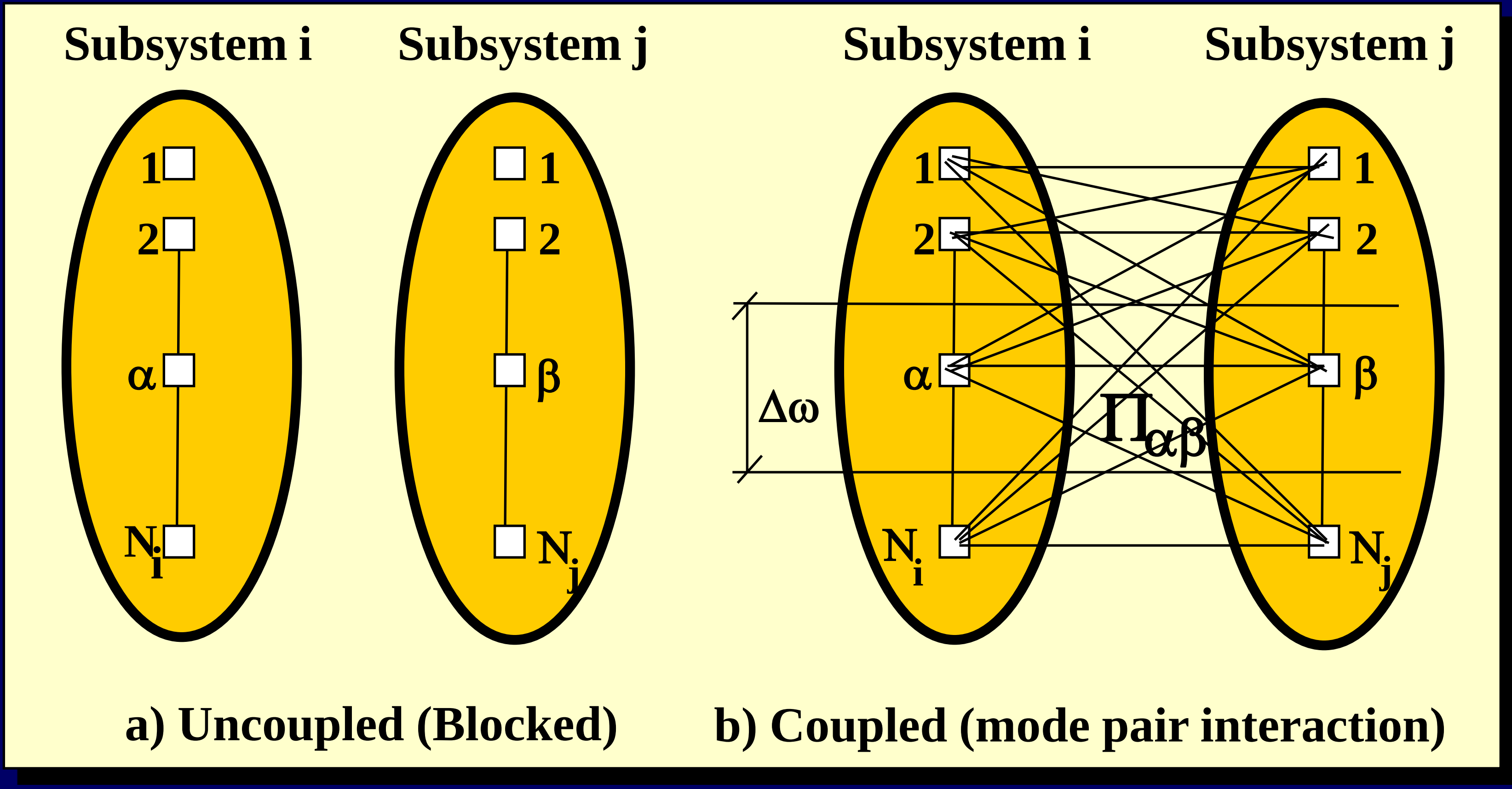
- Example of vibro-acoustic frequency response function scatter for 99 identical cars (Kompella et al, SAE 1995)
- Carefully controlled measurements
- Can be shown to be valid for any structure for modal overlap > 2-3 and very small variation in material or geometrical parameters

# Statistical Energy Analysis



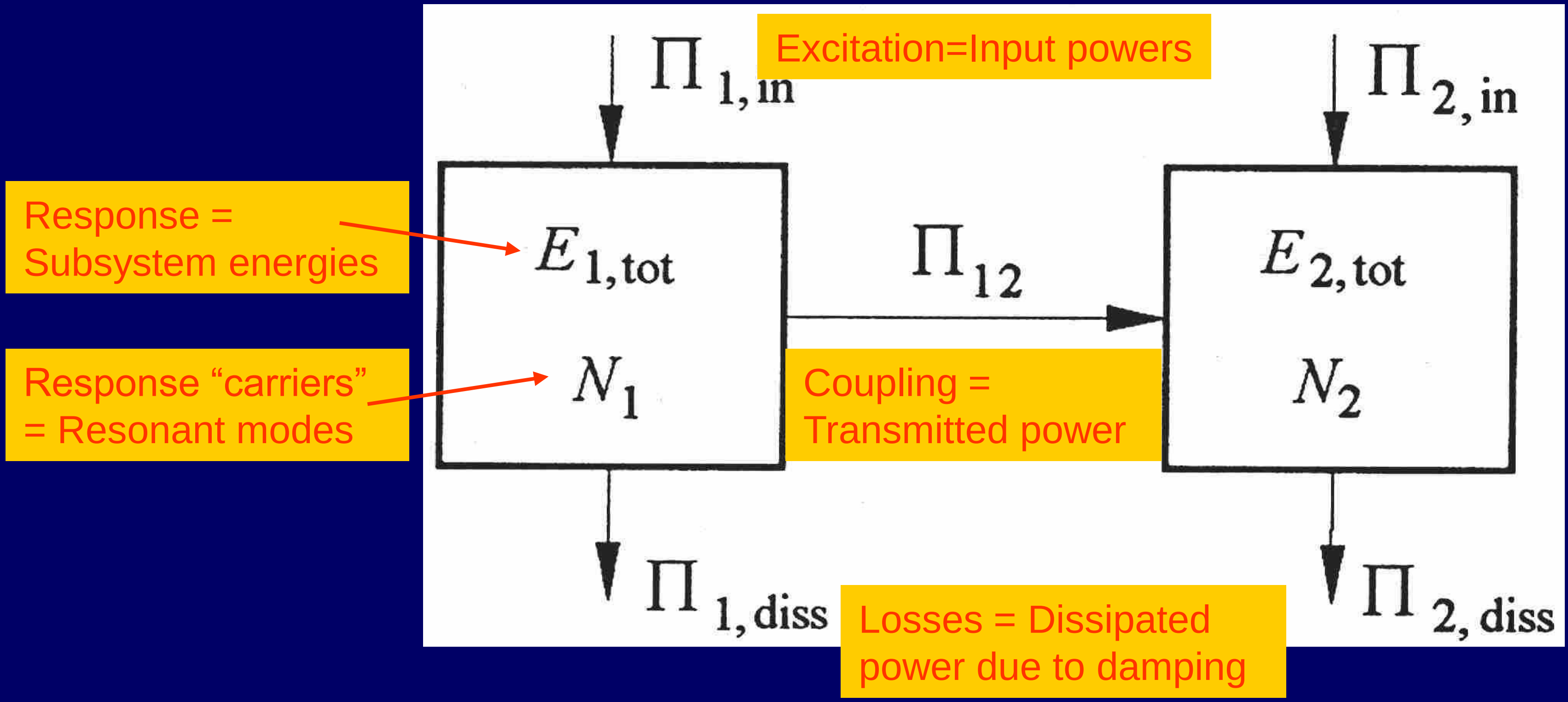

**Mode count in typical ship structure**

# Coupling Between Multi-modal Subsystems

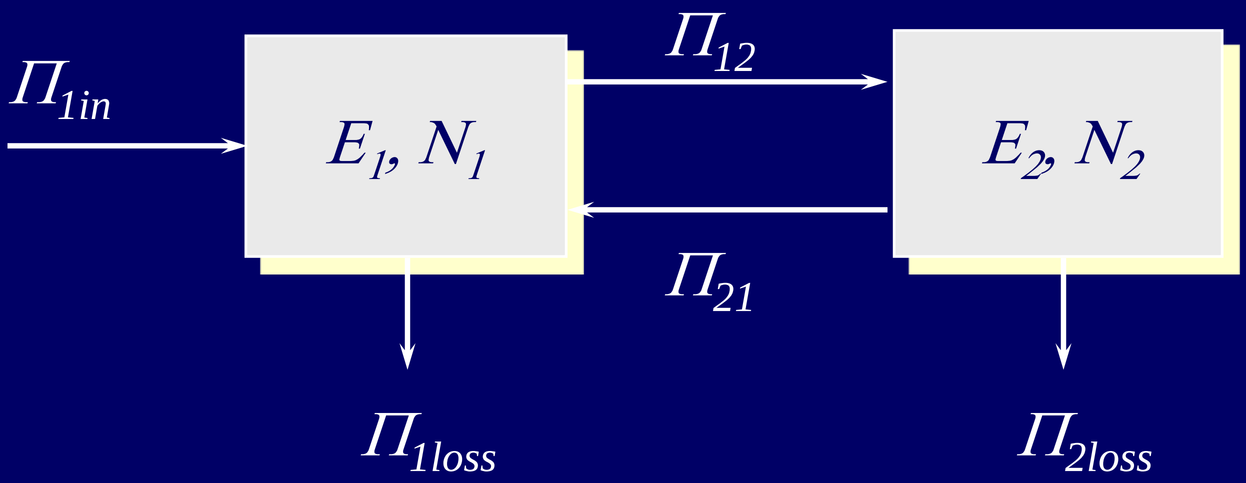


# Statistical Energy Analysis

SEA applied to a system with two coupled subsystems (e.g. bending waves in two plates):



# Energy balance equation - 2 subsystems




$$\Pi_{1in} = \omega\eta_1 E_1 + \omega\eta_{12} E_1 - \omega\eta_{21} E_2 =$$

$$\omega\eta_1 E_1 + \omega\eta_{12} N_1 \left(\frac{E_1}{N_1} - \frac{E_2}{N_2}\right)$$

# SEA Hypothesis - Basic Assumptions

$$\Pi_{ij} = \Delta N_i \sum_{\alpha,\beta} g_{\alpha\beta}(E_{m,i} - E_{m,j}) = \omega\eta_{ij}\Delta N_i(E_{m,i} - E_{m,j})$$

**IF:**

- oscillators of set 1 are *weakly coupled* to oscillators of set 2
- all generalized *modal forces* are *uncorrelated*
- natural frequencies are *uniformly probable* over a frequency interval $\Delta\omega$ ($\rightarrow$ subsystem ensembles)
- oscillators in each set have *equal energies*
- total subsystem energy = energies of *resonant modes only*

# SEA Model Parameters

- ***Dissipative loss factors*** **for subsystems - $\eta_i$**
- ***Input powers*** **to subsystems - $\Pi_i$**
- ***Modal densities*** **for subsystems - $n_i$**
- ***Coupling loss factors (CLFs)*** **between subsystems - $\eta_{ij}$**

# Energy balance equations, N subsytems

$$\omega \cdot \begin{bmatrix} (\eta_1 + \sum \eta_{1i}) n_1 & -\eta_{12} n_1 & -\eta_{13} n_1 & \cdots & -\eta_{1N} n_1 \\ -\eta_{21} n_2 & (\eta_{21} + \sum \eta_{21i}) n_2 & -\eta_{213} n_2 & \cdots & -\eta_{21N} n_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -\eta_{N21} n_N & -\eta_{N2} n_N & -\eta_{N3} n_N & \cdots & (\eta_N + \sum \eta_{Nii}) n_N \end{bmatrix} \cdot \begin{Bmatrix} E_1 / n_1 \\ E_2 / n_2 \\ \cdots \\ E_N / n_N \end{Bmatrix} = \begin{Bmatrix} \Pi_{1in} \\ \Pi_{2in} \\ \cdots \\ \Pi_{Nin} \end{Bmatrix}$$

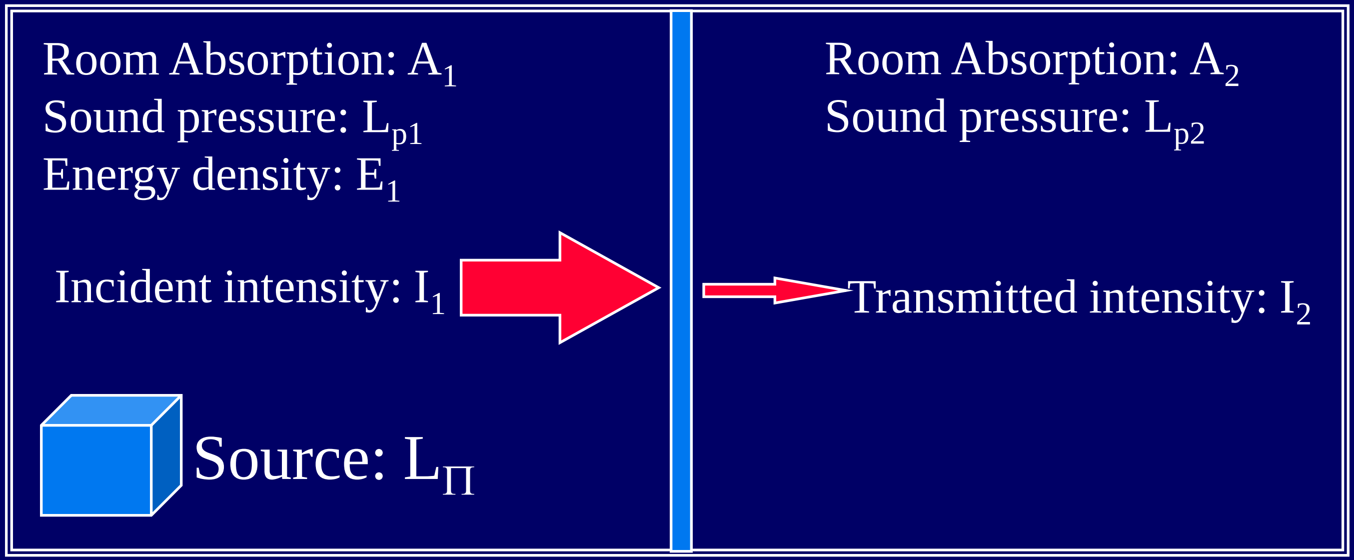
Or in matrix notation:

$$\omega [C] \left\{ E_m \right\} = \{ \Pi \}$$

# SEA = Well known applied acoustics?

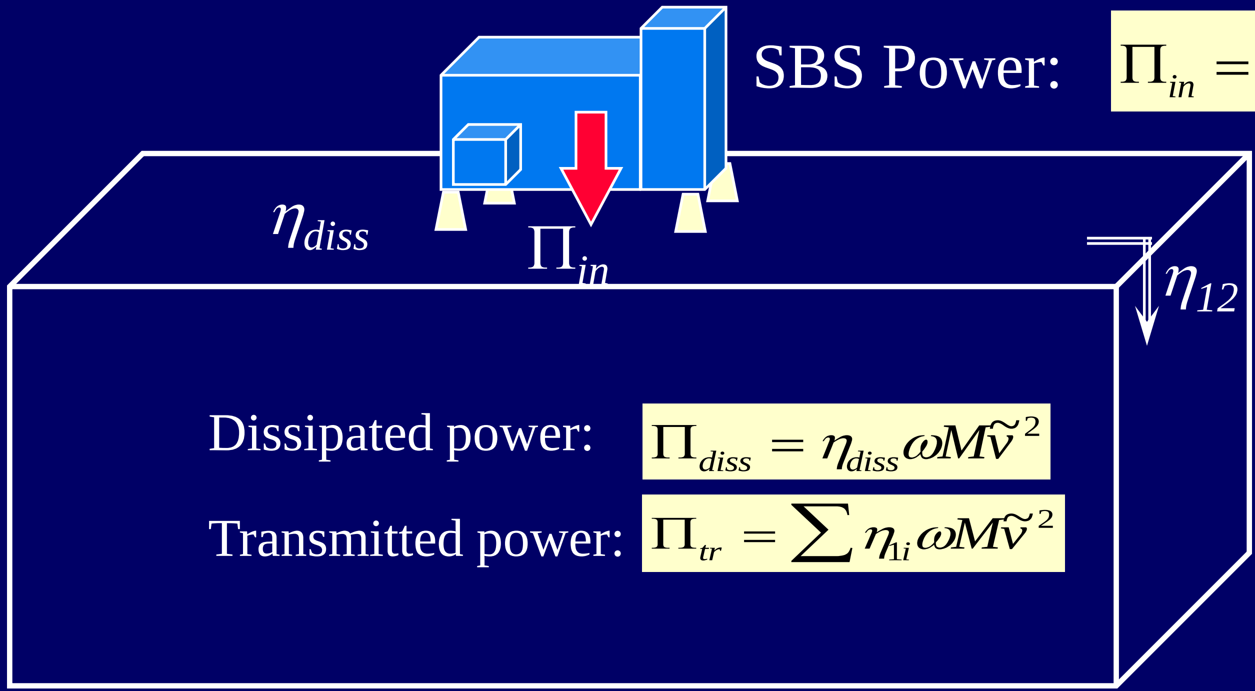
- **Statistical room acoustics = SEA:**
  - Diffuse field concept
  - Room absorption concept
  - Sound power of sources → Energy density → SPL
  - Sound isolation concepts (Transmission Loss for wall etc)
- **Radiation from panels = SEA:**
  - Radiation efficiency concept
- **Structure borne sound transmission = SEA:**
  - Transmission coefficients of junctions etc
  - Power transmission via vibration isolators
  - Damping, damping treatments

# Example 1 - Sound transmission between rooms




$$TL = 10\log(I_1/I_2) \quad \text{dB}$$

# Example 2 - Vibration transmission from source




SBS Power: $\Pi_{in} = \mathrm{Re}\left[Y_{in}\right] \cdot \tilde{F}^2$

Dissipated power: $\Pi_{diss} = \eta_{diss} \omega M \tilde{v}^2$

Transmitted power: $\Pi_{tr} = \sum \eta_{1i} \omega M \tilde{v}^2$

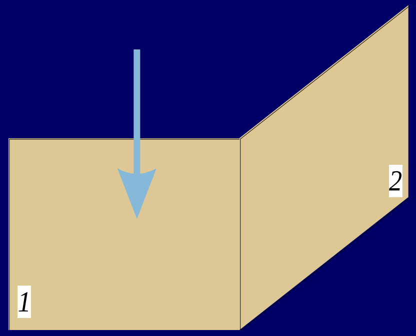
Power balance: $\Pi_{in} = \Pi_{tr} + \Pi_{diss}$

# Types of application

- **Response prediction**
  - Model parameters are known
  - Input powers are known into each subsystem.
  - Energies of all subsystems are calculated according to :

$$\{E\} = \frac{1}{\omega} \cdot [\eta^o]^{-1} \cdot \{P\}$$

  - Example two-subsystem model




$$E_1 = \frac{1}{\omega \cdot \eta_1} \cdot P_1$$

$$E_2 = \frac{\eta_{12}}{\omega \cdot \eta_1 \cdot \eta_2} \cdot P_1$$

# Types of application

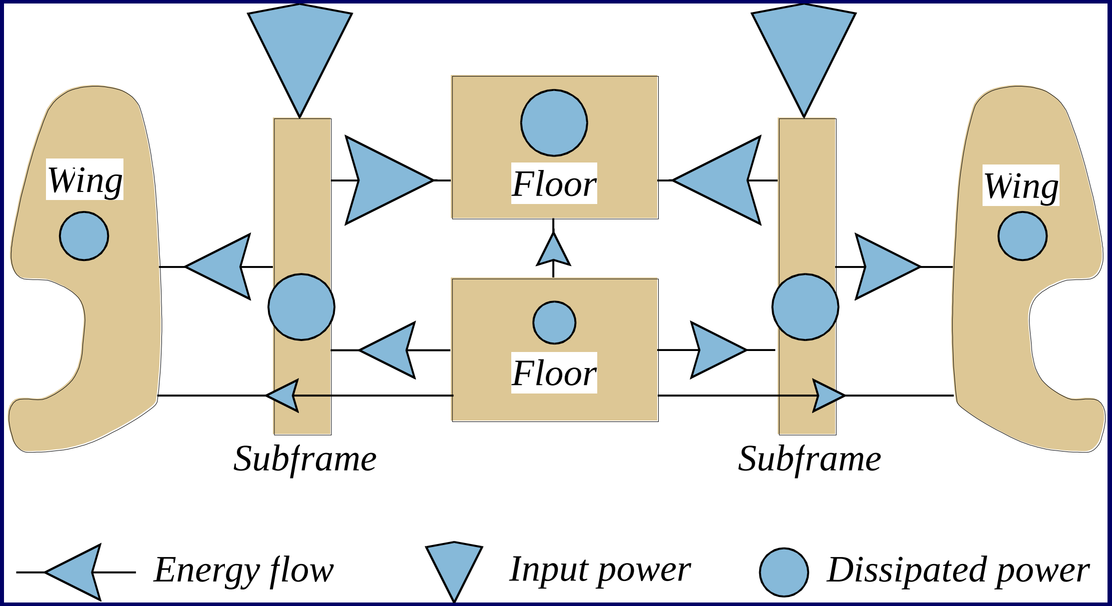
- **Calculation of power input (source localization).**
  - Model parameters are known.
  - Energy levels are known (from measurements).
  - Power input to all subsystems are calculated as:

$$\{P\} = \omega \cdot [\eta^o] \cdot \{E\}$$

# Types of application

- Energy flow within structures:
  - –Known: model parameters, energy or input power levels.
  - –Power flow between subsystems are calculated as:

$$P_{ij} = \omega(\eta_{ij}E_i - \eta_{ji}E_j)$$



# SEA - Advantages and weaknesses

- **Advantages of SEA.**
  - SEA employs a small number of degrees of freedom : energy / subsystem.
  - It only requires a relatively coarse idealization of the physical system.
  - An SEA model allows an analyst to retain a “feel” for the behaviour of the system in terms of few physical properties : computer runs are cheap and quick.
  - If forms an excellent framework for designing and provides the designer with a model on which to base strategies for ”robust” modification of system responses.

# SEA - Advantages and weaknesses (cont.)

- SEA is based on conservation of energy. Violations can easily be detected.
- SEA model can be employed to detect and quantify sources (source localization).
- SEA model can be used to evaluate the main vibrational transmission paths in structures.

## **Weaknesses of SEA.**

- Model relevance and accuracy depends on choice of subsystems. No established rules exist.
- SEA considers only reverberant energy of vibration which is uniformly distributed : not fulfilled for highly damped subsystems.

# SEA - Advantages and weaknesses

- Energies can not be directly measured.
- SEA produces global results/subsystem. No local information is available (advantage for high modal overlap)
- Tonal and narrow band is difficult to handle.
- SEA results are more unreliable if only few resonant modes are present in a band or if modal overlap is low.
- SEA gives unconservative estimates of vibration transmission along chain like subsystems.
- SEA requires experienced analysts (as does qualified dynamic FEM/BEM modelling).