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VIBROACOUSTIC DESIGN OF PRODUCTS. HOW ACCURATELY CAN WE PREDICT SOUND TRANSFER PROPERTIES?

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1 BACKGROUND

Significant variations are obtained in vibro-acoustic transfer properties between individual products that are produced to be identical. Variations in the order of ± 5-8 dB for transfer functions are common in serial production of road vehicles, aircraft, ships, appliances etc.

Variation of transfer function characteristics between vehicles with identical design has been reported in [1]-[3]. Kompella et. al. presented measured frequency response functions (FRFs) for a large number of vehicles. The FRFs show more randomness and larger scatter as frequency increases, see Figure 1. This is usually considered to be due to low quality of components or assembly, and QA-programs are introduced to reduce the variability. Highly detailed dynamic FE-modelling is also justified by this ”low quality” assumption.

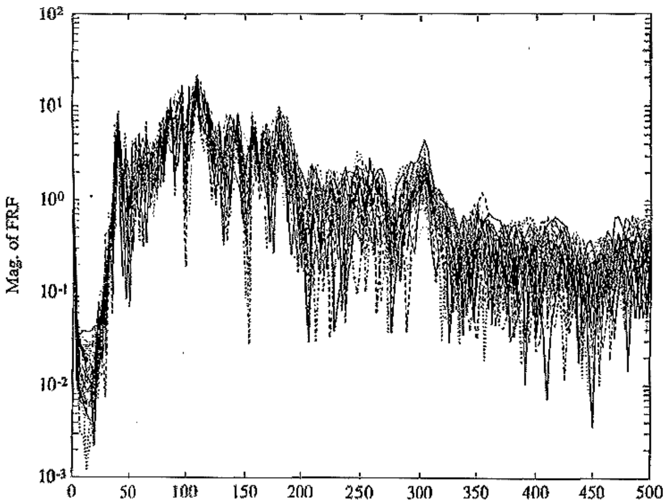

Fig. 1: Magnitudes of the 99 structure-borne FRFs for the RODEOs for the driver microphone [1].

The relevance of detailed predictions with deterministic methods can be questioned at frequencies with significant modal overlap. Resources for creation and experimental updating of the models must be optimised with respect to achievable prediction accuracy. Statistical energy methods(SEA, EFA etc.) may be efficient alternatives at medium and high frequency, with much lower modelling and computation effort. Variability due to quality or tolerance problems will not be addressed here. Instead the basic limitation of deterministic prediction of dynamic response for multi-modal systems due to input parameter uncertainty is demonstrated.

A good summary of the fundamental limiting factors for deterministic modelling and analysis was presented in a recent SEA review paper by Fahy [4]. In addition to fact that

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FE- or BE-models become very large at higher frequency due to the finer meshing and that the modelling effort is much increased due to more attention to geometrical detail, the following fundamental predictability limits for prediction of the response in detail exist:

- *Uncertainty about precise dynamic properties*. Sensitivity of eigenfrequencies and phase to boundary conditions, thickness- and damping distribution etc. increases with mode order.
- *Modal summation.* Contributions from an increasing number of modes are added at each frequency as frequency and/or damping increases.
- *Uncertain dynamic properties of joints*. Dynamic force transmission properties of joints are not well defined especially uncertain at higher frequencies.
- *Uncertain material properties.* Properties of alloys, polymers and composite materials are much harder to predict and will vary much more due to temperature, static loads etc.
- *Uncertain modal damping estimation*. For detailed deterministic prediction, the correct spatial damping distribution or individual modal damping has to be applied.

Response prediction of multi-modal systems is therefore a probabilistic problem. High-frequency response of a product shall be described by an ensemble-average behaviour, with statistical estimation of the distribution of responses around this average. One may randomise parameters and properties using assumed distributions and use deterministic FE-computations (Monte-carlo simulation). An alternative is statistical energy methods (SEA) [4], [5].

2 THEORY

The statistics of multi-modal systems was first derived for room acoustics, see e.g., [7]-[9]. It has also been studied during the SEA development [10]. Schröder derived some fundamental results already in 1954 [7]. One considers a system where the response at each point and frequency is determined by the sum of a sufficient number of modes with random phase, and where no individual mode is dominating the sum. The real and imaginary parts of the complex response function are assumed to have a gaussian distribution[7]:

$$W(\mathrm{Re}(x)) = \frac{1}{\sqrt{2\pi\overline{\mathrm{Re}(x)^2}}}\, \mathrm{e}^{-\mathrm{Re}(x)^2 / 2\overline{\mathrm{Re}(x)^2}} \quad \text{and} \quad W(\mathrm{Im}(x)) = \frac{1}{\sqrt{2\pi\overline{\mathrm{Im}(x)^2}}}\, \mathrm{e}^{-\mathrm{Im}(x)^2 / 2\overline{\mathrm{Im}(x)^2}} \qquad (1)$$

The gaussian distribution of real/imaginary parts is valid when the response is given as the sum of several complex independent (modal) vectors, of which no one is dominating, see Figure 2a. The standard deviation of the FRF amplitude is derived to σ = 5.57 dB [7] in this case. Figure 2b shows a FRF with a dominating component, e.g., a dominating mode or the direct wave field of a damped system. For this case the gaussian distribution does not apply.

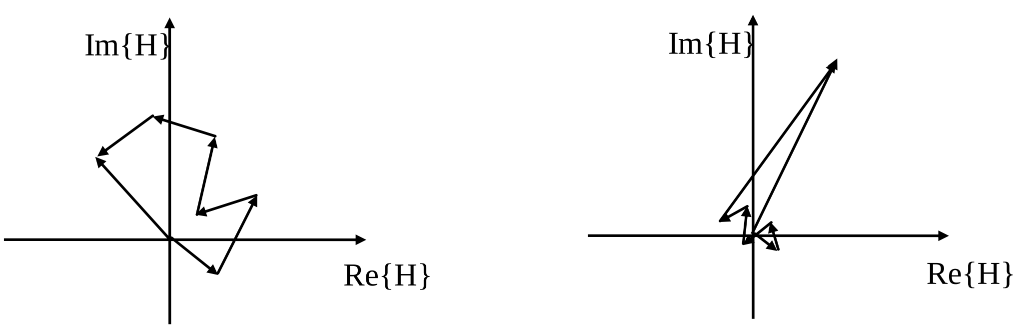

Fig. 2: The complex FRF and its modal components, for one frequency. a) No mode dominates. b) One component is dominating.

3 GENERIC VIBRO-ACOUSTIC MODELS

The theory above was derived generally from a summation of complex vectors. It should be valid for any dynamic system for which the FRF can be expressed as such a sum. The FRF for a generic system with N vibrational modes is

$$H(\boldsymbol{x},\boldsymbol{x}_e,\omega)=\frac{\bar{v}(\boldsymbol{x},\omega)}{F(\boldsymbol{x}_e,\omega)}=\frac{4\,\mathrm{j}\omega}{M}\sum_{i=11}^{N}\frac{\phi_i(\boldsymbol{x})\cdot\phi_i(\boldsymbol{x}_e)}{\omega_i^2(1+\mathrm{j}2\zeta_i)-\omega^2} \qquad (2)$$

where $\boldsymbol{x}$ is the spatial vector, $\boldsymbol{x} = [x,y,z]$, index e refers to excitation point, M is total mass, ω is excitation frequency and ϕ_i is the eigenfunction of mode *i*.

A dynamic system represented by a sum of ”modes” according to Equ. (6) for which the eigenfrequencies are distributed as $\omega_i = 200\pi\cdot\log(i+1)$ and the eigenfunctions at excitation and response positions are random numbers between -1 and 1 is used. Random shifts in eigenfrequencies and damping for the individual modes are introduced to simulate parameter variations for real structures. The eigenfrequencies are shifted as

$$\omega_{ij}=\omega_{ij0}\cdot(1+\varepsilon U) \quad \text{or} \quad \omega_{ij}=\omega_{ij0}\cdot(1+\varepsilon U_{ij}) \qquad (3a, b)$$

where ω_{ij0} is the unshifted eigenfrequency, ε is the amplitude of the random variation and U and U_{ij} are random numbers with normal distribution (m=0, σ=1). For *global* parameter variations (e.g. average plate density or modulus) eigenfrequencies are shifted with the same relative frequency εU (Equ. 3a). Examples are given in [6]. *Local* variations of thickness, mass, boundary conditions etc. result in individual shifts in eigenfrequency for each mode (Equ. 3b), where each ω_{ij} is shifted by εU_{ij}.

The modal damping has the same nominal value, ζ_{ij0}, for all modes. The uncertainty in damping is modelled by an exponential normal distribution, see equation (4). U_{ij} has a normal distribution with an mean value of 0 and a standard deviation of 1. An exponential normal distribution is chosen as it provides a realistic damping distribution for the modes.

$$\xi_{ij}=\xi_{ij0}\cdot 10^{\varepsilon U_{ij}} \qquad (4)$$

Figure 3 shows the difference in FRF obtained for two samples of the generic model. The modal overlap factor, which is defined as

$$MOF=n(f)\eta f \qquad (5)$$

where *n(f)* is the average modal density (modes/Hz) and η the loss factor at frequency *f*, is larger than 1 for *f* > 100 Hz in this case. Schröder's formulation is applicable when the modal overlap factor is larger than ≈2-3. When the modes have approximately equal excitation no single mode will dominate the response in that case and the response is determined by a sum of several modes with different phase and amplitude.

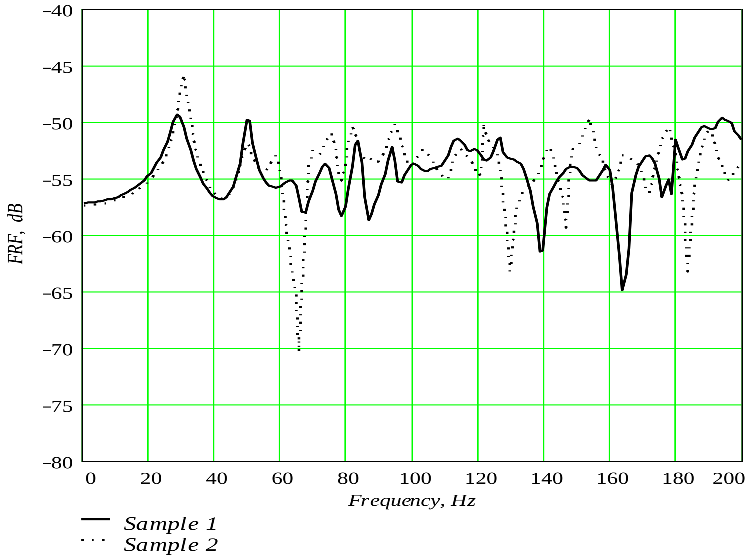

Fig. 3: FRFs for the generic modal expansion model with η = 5%. Standard deviations: 3% for eigenfrequencies, 30 % for logarithm of the modal damping .

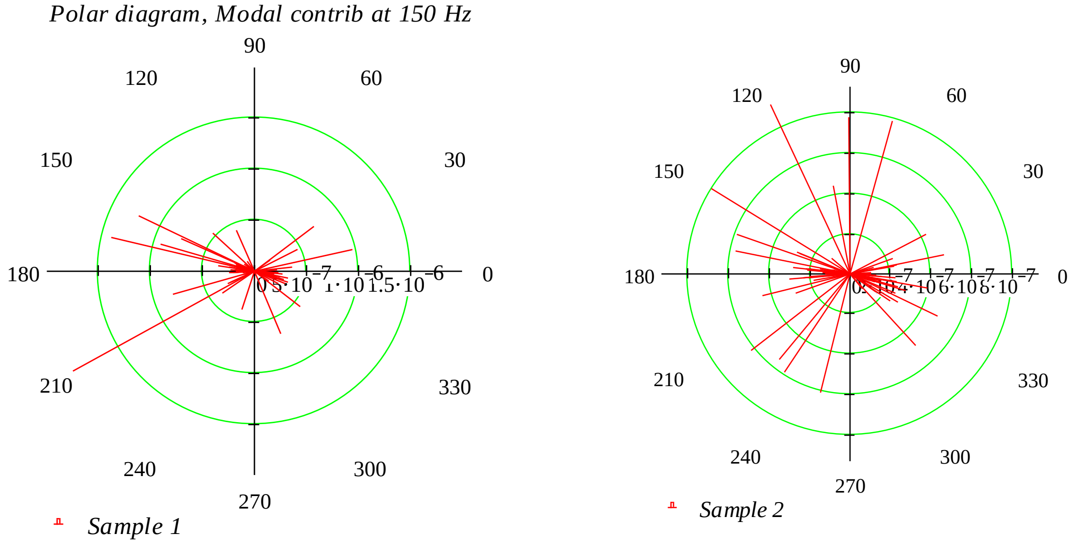

Fig. 4: Modal contributions at 150 Hz for the two samples from the generic modal expansion model.

The contributions from individual modes are shown for 150 Hz in Figure 4 a and b for the two samples. The large differences in phase and amplitude of modal contribution vectors are obvious.

A thin, rectangular plate was used to exemplify a real multi-modal component [6]. The dimensions of the plate are given in table 1. The plate is simply supported and excited at one point.

The analytic model of the plate was used to calculate the variation of FRFs resulting from both global parameter and from localised parameter variations [6]. Only some examples of the later will be shown in this paper. The excitation and response positions on the plate are arbitrarily chosen but the same for all plate samples so the same point-point frequency response function is calculated for all plate samples.

	Plate
l_x [m]	0.6
l_y [m]	1.67
l_z [m]	10^{-3}
c_0 [m/s]	-
E [Pa]	$2.10 \cdot 10^{11}$
n [-]	0.3
r [kg/m^3]	7850
ξ [-]	0.03

Table 1: Input parameters for the rectangular plate

Local scatter in geometry, thickness, pre-stresses and boundary conditions will introduce individual shifts of eigenfrequencies based on how the variations correlate with mode shapes. Uncertainties in material and joint damping distribution, sound radiation etc. result in variations in individual modal damping factors, modelled as randomly distributed damping for individual modes. This may cause largely varying transfer functions as shown in [6]. The necessary variation of the input parameters of the rectangular plate to get randomly varying FRFs for modal overlap larger than 2-3 is, e.g., a 2% eigenfrequency variation combined with a 20% variation in the logarithm of damping for the studied plate, see Figure 5.

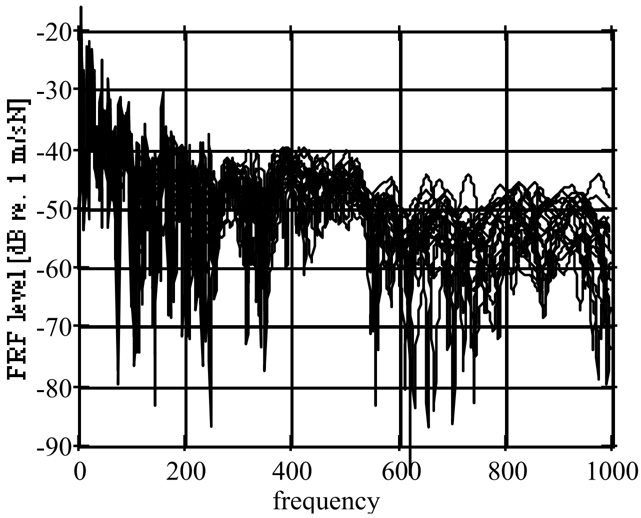

Fig. 5: FRFs for combined random variations of eigen-frequencies and modal damping. Standard deviations: 2% for eigenfrequencies and 20% for the logarithm of damping factor.

Energy methods (SEA) [10] calculate spatial average responses for the subsystems instead of the response at specific points on the system. However, as shown above, detailed FRF estimation is of limited value, since small input parameter uncertainties will lead to low precision in the prediction anyway.

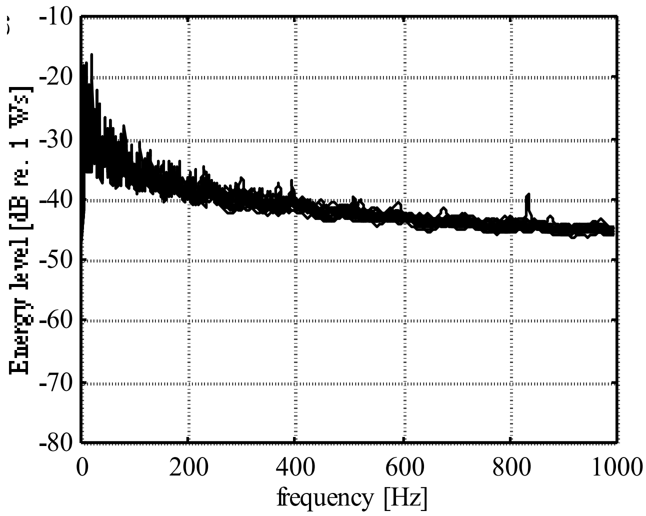

Fig. 6: Variation of spatial average velocity level (proportional to the kinetic energy) between different samples of the plate. Same variation of parameters as Fig 5.

The spatial average energy response should fluctuate much less than the FRFs for corresponding variations in eigenfrequency and modal damping. The energy response of the same plate as before was calculated using the analytical model developed by Fredö [13]. The same variation of eigenfrequencies and damping results in Figure 6. As expected, the energy response levels show much less variation than the FRFs at frequencies where modal overlap is significant.

4 SCATTER IN BUILT-UP STRUCTURES

The energy flow model developed by Fredö [13] for two connected plates was used to investigate two connected plates in a L-configuration as illustrated in Figure 7. The parameters for the configuration are given in Table 2.

Fig. 7: The L-configuration of two simply supported plates used in the analytical model by Fredö [13].

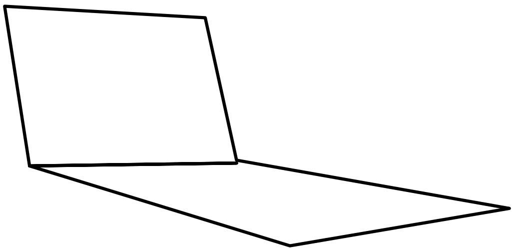

	Plate 1	Plate 2
l_{side} [m]	0.6	0.84
$l_{junct.}$[m]	1.67	1.67
h [m]	10^{-3}	10^{-3}
E [Pa]	$210 \cdot 10^9$	$210 \cdot 10^9$
ν [-]	0.3	0.3
ρ [kg/m^3]	7850	7850
ξ [-]	0.03	0.03

Table 2: Parameters of the L-plate configuration.

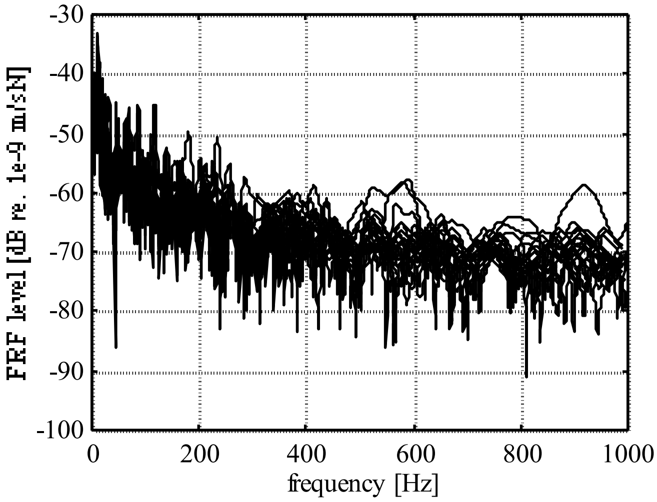

Fig. 8: Variation of FRF levels between points on two subsystems, calculated for the L-plate. Combined local parameter variations. Standard deviations: 2% for eigenfrequencies and 20% for logarithm of modal damping.

The result of combined variations of individual modal eigenfrequencies and damping, same as those used for the simple plate, is shown in Figure 8. The FRFs between an arbitrary point on plate 1 and a point on plate 2 is shown. The standard deviation in the frequency range with significant modal overlap reaches the same 5-6 dB value as for the single plate. The calculated response energy level in plate 2 with excitation in plate 1, corresponding to the result shown in Figure 6 for the single plate, is given in Figure 9.

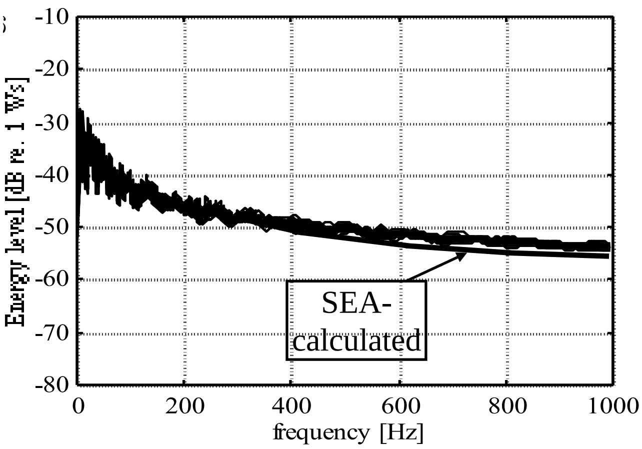

Figure 9: Variation of spatial average velocity level in the receiving plate of the L-plate. Combined local parameter variations with standard deviations: 2% for eigenfre-

quencies and 20% for logarithm of damping.

The same response energy level has also been calculated with Statistical Energy Analysis (SEA). The plates were modelled as bending wave subsystems only using the AutoSEA software. The result has been included in Figure 9 for comparison. As can be seen, the agreement with the exact analytical result is quite good. The SEA modelling, computations and plotting of resulting energy levels or response levels were performed in about 5-10 minutes. Most of this time was spent on input of the subsystem data from Table 2 above.

5 EXAMPLES OF PRACTICAL PARAMETER TOLERANCES

Some work has been done by the author and his colleagues at Ingemansson as well as Chalmers on investigating actual component eigenfrequency and FRF variation. It has been carried out as parts of confidential industry related projects and details can not be quoted here. However, in general terms, even stamped, thin steel plate components without attached hardware will often show a noticeable FRF standard deviation (1-3 dB) in the frequency range of modal overlap > 2-3. Considering the small manufacturing tolerances and a well controlled material, this is rather surprising.

The experimental investigations needed utmost care in order to obtain repeatable edge conditions and shaker attachment. These could otherwise influence the measured variability of the FRFs as well as the scatter in natural frequencies considerably.

For cast iron and cast aluminium as well as polymer components, the FRF standard deviation in the modal overlap region may approach the 5 dB upper limit for average production and material tolerances.

FE-models may also be used to investigate how specific parameter tolerances influence the scatter of natural frequencies. One example of simulating local thickness (σ_t = 10%), Young´s modulus (σ_Y = 5%), and density (σ_d = 5%), variations for a plate is given in Figure 10. The calculations were made for a rectangular, simply supported plate using 300 triangular elements. The parameters have been varied randomly for 40 patches. The actual influence on natural frequencies when varying these parameters will start to diminish when the size of the ”patches” become smaller than the correlation length of the mode, which is approximately half a bending wavelength.

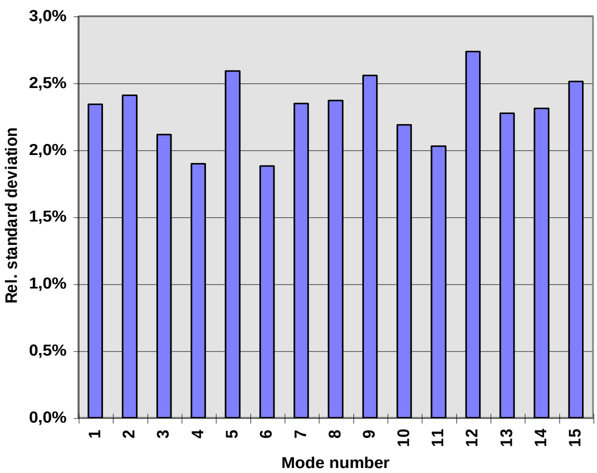

Fig. 10: Standard deviation for natural frequencies of different modes of the rectangular thin plate, obtained with FEM.

6 CONCLUSIONS

Frequency response functions for simple as well as built-up vibro-acoustic systems with overlapping modes are quite sensitive to small variations in local input parameters. Numerical simulation using realistic parameter tolerances show that the FRFs may become well randomised already when the modal overlap factor reaches about 2-3. Experimentally obtained variability by the author and published by other workers for nominally identical products like cars compare well with the simulations presented here and in [6].The standard deviation for FRFs between specific points at a specific frequency is considerable. The variance is substantially smaller for spatial RMS-averages of the FRFs, which correspond to subsystem response energy quantities, as they are used in existing statistical energy analysis (SEA) prediction.

The reliability of deterministic response prediction in road vehicles, aircraft, spacecraft etc. at medium and high frequency is not primarily determined by the size or the geometrical detail of a FE-model or even the modelling skill of the analyst. The limit is set by input parameter accuracy requirements as small variations in eigenfrequency and damping of individual modes will produce large FRF scatter due to overlapping modes. Updating of the FE/ BE-model against hardware will not reduce this random error due to scatter betveen individual products.

Statistical energy methods (e.g. SEA) for prediction are therefore serious alternatives to deterministic modelling at medium and high frequency for many products. The modelling effort and computing resources are much less. The results also directly represent the average response for a random ensemble of structures. Result from a deterministic model may erroneously be interpreted as accurately representing the entire ensemble of products due to the detailed information content, especially if the model has been carefully updated.

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